Non-Dispersive Accelerated Wave packets

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Layout

- Matter-wave interaction
- Atom Optics Fermi Accelerator
- Quantum Recurrences
- Dynamical Localization
- Mater waves Coherent Acceleration
- Non-Dispersive Accelerated Atom

Johannes Kepler (1571-1630)

- As the comet comes near the Sun, the Sun's radiation energy pushes the tail behind the comet causing the tail to grow. Its tail is pushed in front when the comet moves away from the Sun.

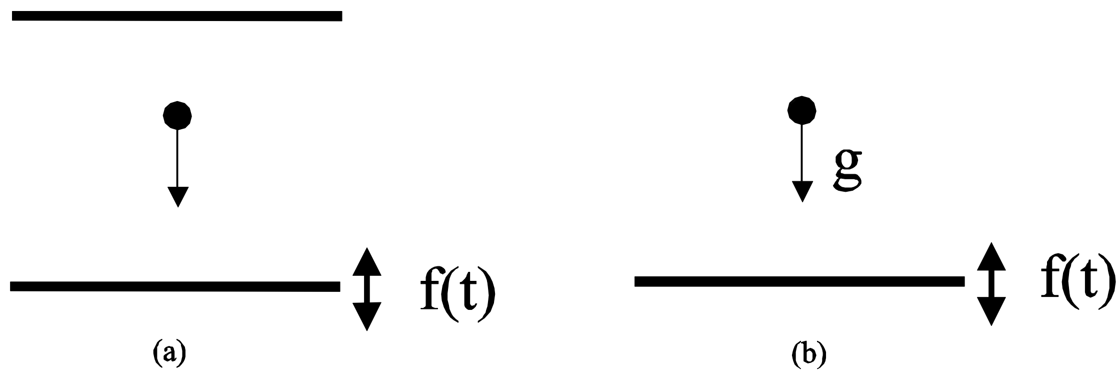
On the Origin of the Cosmic Radiation

ENRICO FERMI
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magmetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

PHYSICAL REVIEW VOLUME 75, NUMBER 8 APRIL 15, 1949

Fermi Accelerator



Fermi-Ulam Accelerator

Fermi-Pustylnikov Accelerator

Kazantsev (1974), Cohen-Tannoudji
Interaction of an Atom with single mode Field

$$|\Psi\rangle = \psi_g |g\rangle + \psi_e |e\rangle$$

$$H = H_A + H_F + H_{AF}$$

$$H_{AF} = -\vec{d}.\vec{E} = -E_0(\sigma^- + \sigma^+)\vec{d}.u(\vec{r})\cos\omega t$$

- Field intensity is large so that we can treat it classically (semi-classical treatment)
- Field intensity remains the same over the size of the atom (dipole approximation)
- Field frequency is *tuned away* from the transition frequency between any two levels
- Average over faster frequency (rotating wave approximation)

$$i\hbar\frac{\partial\psi_g}{\partial t} = \frac{\hat{p}^2}{2m}\psi_g - \hbar\Omega_R u(\hat{r})\psi_e$$

$$i\hbar\frac{\partial\psi_e}{\partial t} = \frac{\hat{p}^2}{2m}\psi_e - \hbar\delta\psi_e - \hbar\Omega_R u(\hat{r})\psi_g$$

$$\delta = \omega_f - \omega_{eg}$$

$$\Omega_R = \vec{d}.\vec{e}_r E_0 / 2\hbar$$

- Atom stays in its ground state (secular approximation)
- and therefore probability of *spontaneous emission is negligibly small*.

$$\frac{\partial \psi_e}{\partial t} \approx 0; \qquad \frac{\partial \psi_e}{\partial x} \approx 0 \qquad\qquad \psi_e \cong -\frac{\Omega_R}{\delta} u(\hat{r}) \psi_g(t)$$

$$i\hbar \frac{\partial \psi_g}{\partial t} \cong \frac{\hat{p}^2}{2m} \psi_g + \frac{\hbar \Omega_R^2}{\delta} u^2(\hat{r}) \psi_g$$

$$H_{eff} = \frac{p^2}{2m} + \frac{\hbar \Omega_R^2}{\delta} u^2(r)$$

General Hamiltonian for a super-cooled atom in a modulated classical field

$$H_{eff} = \frac{p^2}{2M} + V_0(t)\ u^2(r + \varphi(t))$$

Nonlinear and chaotic
- both in classical and quantum domains

Atomic dynamics in modulated systems

Phase modulated standing wave field $\quad V = V_0 \cos(2kx + \varphi_0 \sin \omega t)$

Amplitude modulated standing wave field : Kicked Rotor Model

$$V = V_0 \cos 2kx \sum_n \delta(t - nT)$$

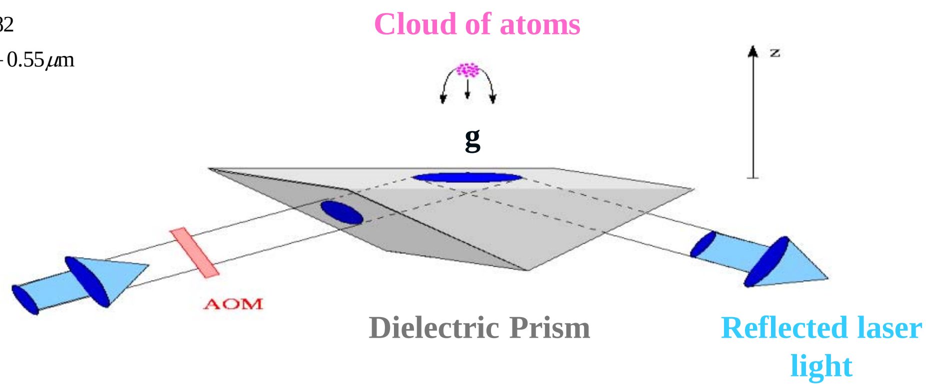
Ion in a Paul trap in the presence of a standing light field

$$V = \frac{1}{2} x^2 (a + b \sin \omega t) + \Omega \cos kx$$

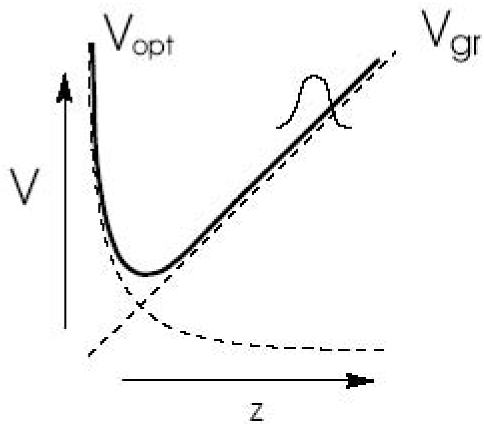
F. Saif, Physics Reports 419, 207 (2005); 425, 369 (2006)

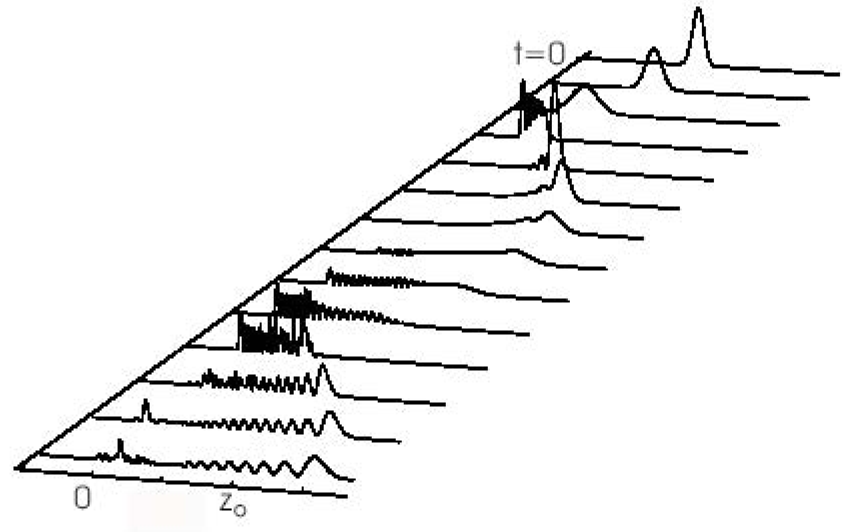
Atomic Fermi-Accelerator

$m = 2.2 \mathrm{x} 10^{-25} \mathrm{kg}$
$\omega = 5.84 \mathrm{kHz}$
$\varepsilon = 0.1 - 0.82$
$\mathrm{k}^{-1} = 0.19 - 0.55 \mu\mathrm{m}$



F. Saif, I. Bialynicki Birula, M. Fortunato, W. P. Schleich, Physical Review A 58, 4779 (1998).





$$\Psi(z)=\frac{1}{\sqrt{\sqrt{2\pi}\Delta z}}\exp\left\{-\frac{(z-z_0)^2}{4\Delta z}\right\}\exp\left\{i\,\frac{p_0 z}{\hbar}\right\}$$

Dimensionless coordinates

$V_0 = \hbar\omega^2\Omega_{eff} / 4mg^2$

$z = \frac{\omega^2}{g} z$

$\kappa = 2kg / \omega^2$

$p = \frac{\omega}{mg} p$

$$\lambda = \omega^2 \varepsilon / 2kg$$

Control Parameters

$\text{dimensionless modulation strength} = \lambda$

$\text{effective Planck's constant} = \lambda\!\!\!^{-} = \frac{\hbar\omega^3}{mg^2}$

$$H = \frac{p^2}{2} + z + V_0 e^{-\kappa(z - \lambda \sin t)}$$

Classical Dynamics of a single particle (Hamilton's Equations)

$$\dot{z} = \frac{\partial H}{\partial p} = p$$

$$\dot{p} = -\frac{\partial H}{\partial z} = -1 + \kappa V_0 e^{-\kappa(z-\lambda \sin t)}$$

Discrete Dynamics of a single particle (Chirikov's Mapping)

$$p_{n+1} = p_n + 4\lambda \cos\varphi_{n+1}$$

$$\varphi_{n+1} = \varphi_n + p_n \quad \mathrm{mod}(2\pi)$$

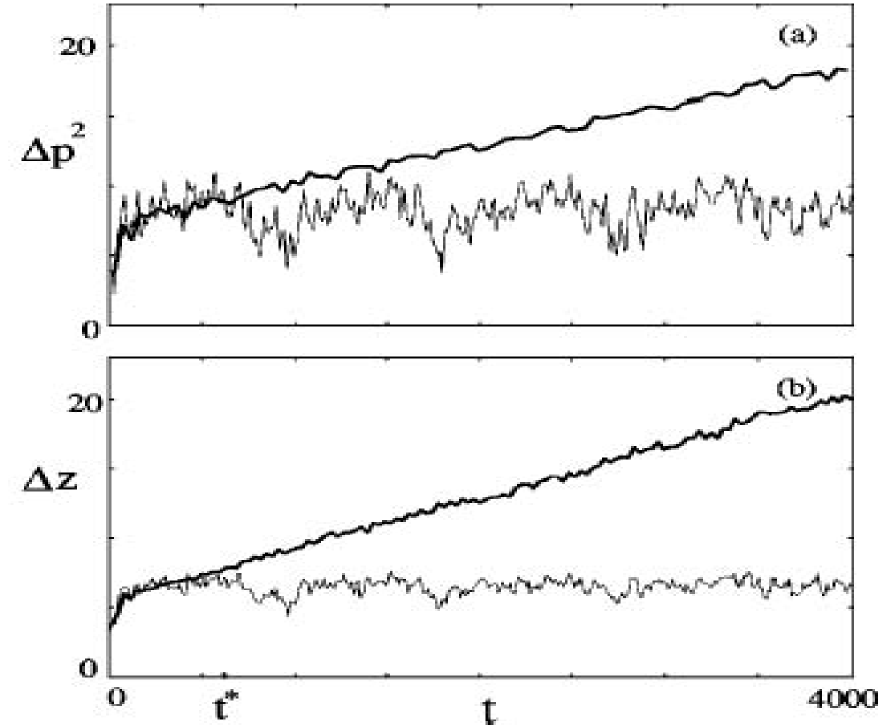
$$D_0 = 8\lambda^2$$

Classical diffusion $\lambda_c = 0.24$

Quantum diffusion* $\lambda_c = \left(\frac{\hbar \omega^3}{4 m g^2} \right)^{1/2} = \frac{\sqrt{\lambda}}{2}$

* F. Benvenuto, G. Casati, I. Guarneri, D.L. Shepelyansky, Z. Phys. B 84, 159 (1991)

In Quantum Case the Width △ z = 2 in position and a corresponding width in momentum △P=Ƙ/2 △z. Here (z0=20, p0=0)



$$\Delta p^2 = 4D_0 t = \Delta z$$

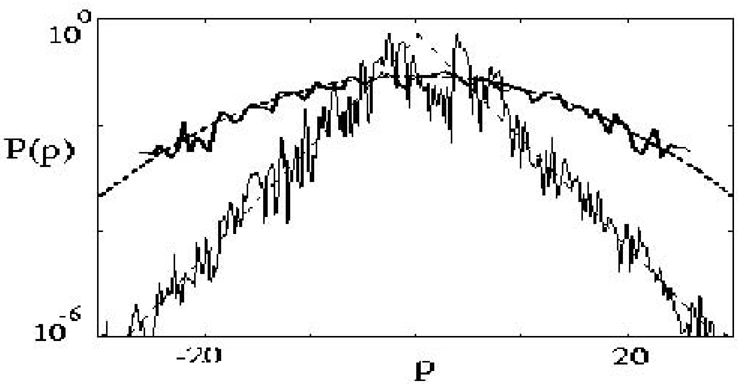
The number of particles = 2000. The height of the exponential potential is V0 = 60, its steepness is K= 0.5, and the modulation strength is¸ λ = 0.5. In the quantum mechanical case the effective Planck's constant is taken as Ƙ =4.

F. Saif, I. Bialynicki-Birula, M. Fortunato, and W. P. Schleich, Phys. Rev. A. 58, 4779, (1998).

Dynamical Localization of Atoms

Momentum Distributions

$$\lambda_c^{(cl)} < \lambda < \lambda_c^{(Q)}$$



Momentum Distributions

$$P^{(cl)}(p) \propto \exp\left(-\frac{p^2}{2k_B T}\right)$$

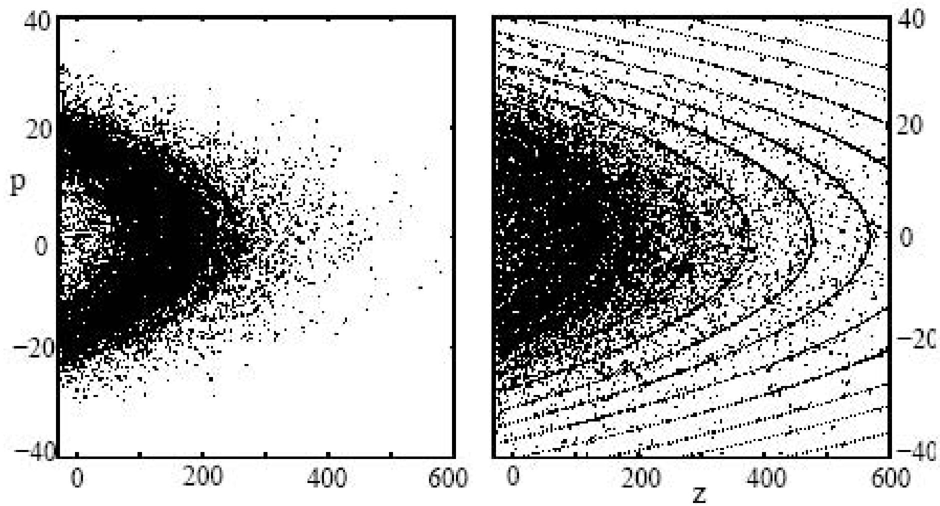
$$P^{(Q)}(p) \propto e^{-l|p|}$$

$\lambda = 0.8 \qquad ƛ = 4$

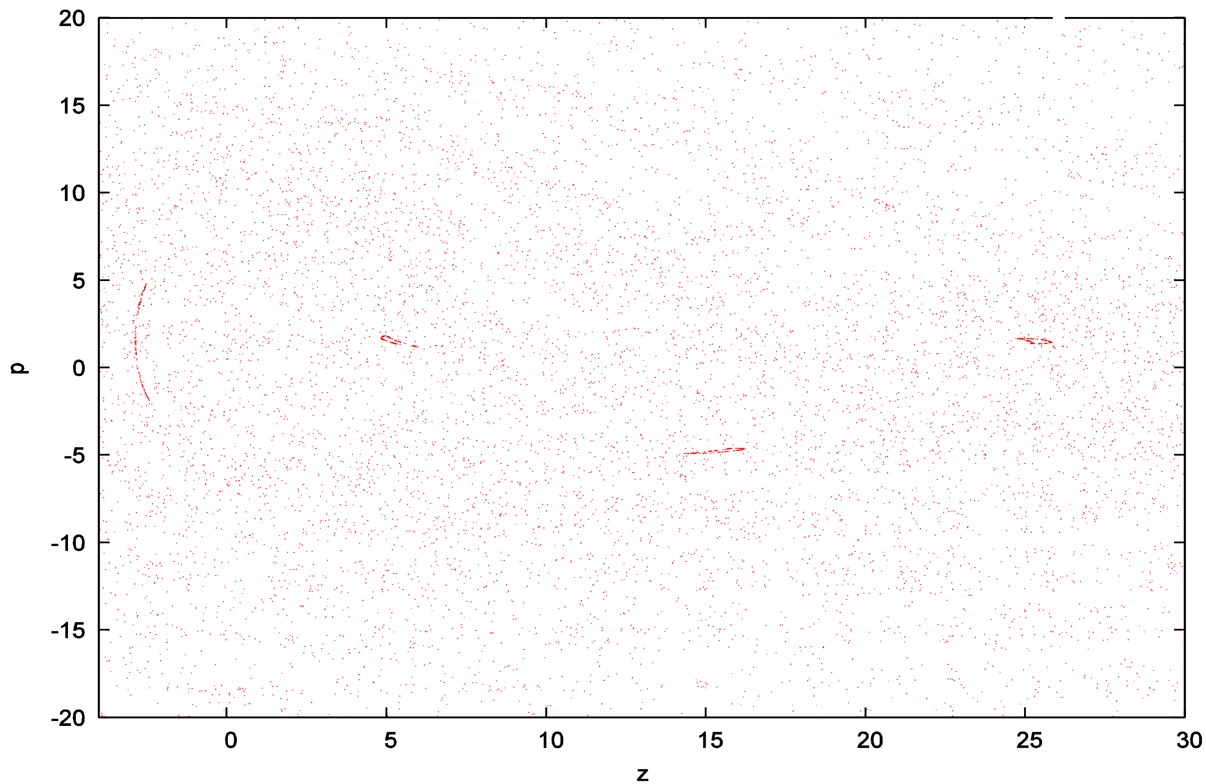
$(z0, p0) = (20,0)$

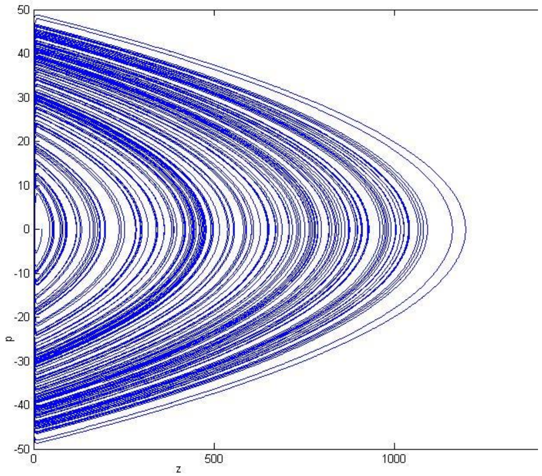
$t = 3200$

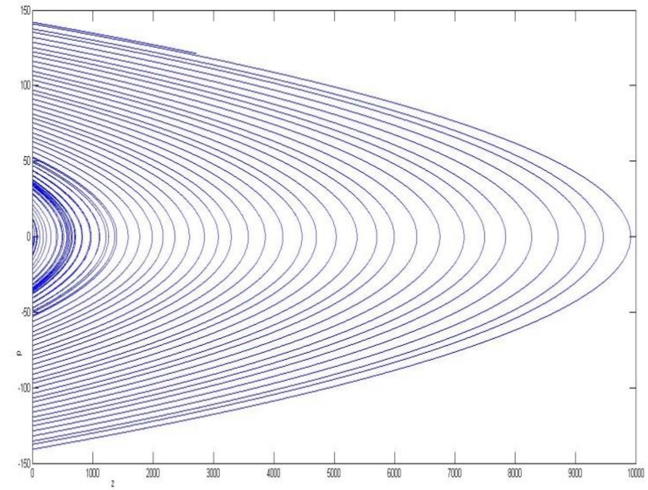
Poincare surface of sections

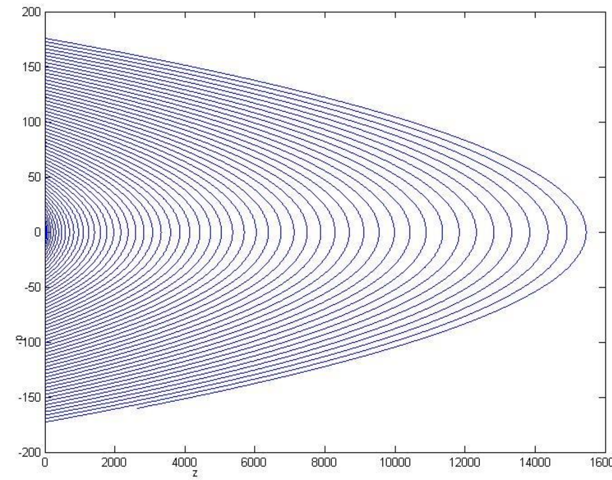


$\lambda = 2.4$ $\lambda = 3.1$


20
15
10
5
0
-5
-10
-15
-20
p
0
5
10
15
20
25
30
z

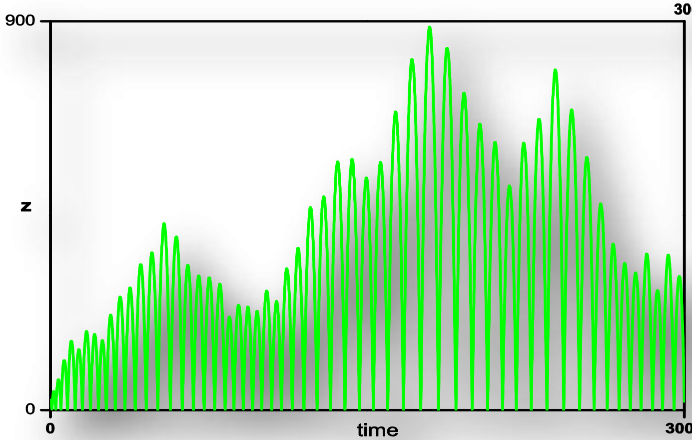

p
z

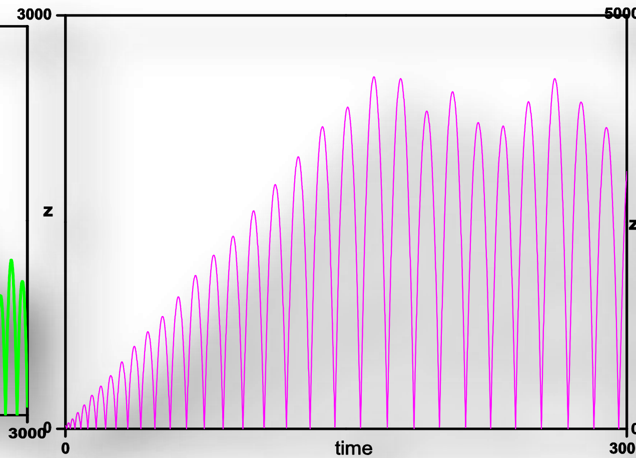

p
z

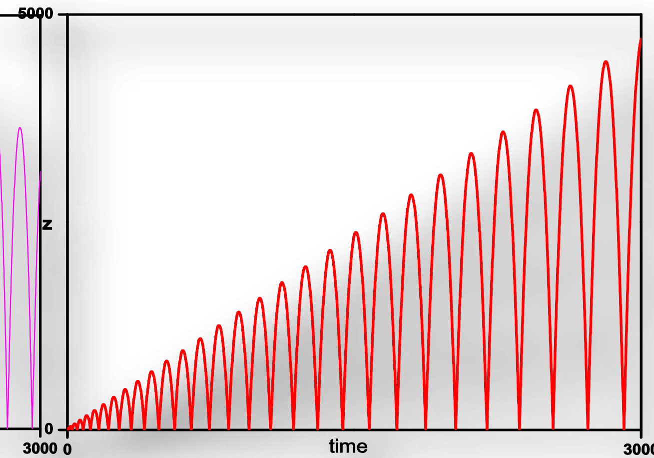

p
z

Classical Analysis. . .

Continued .

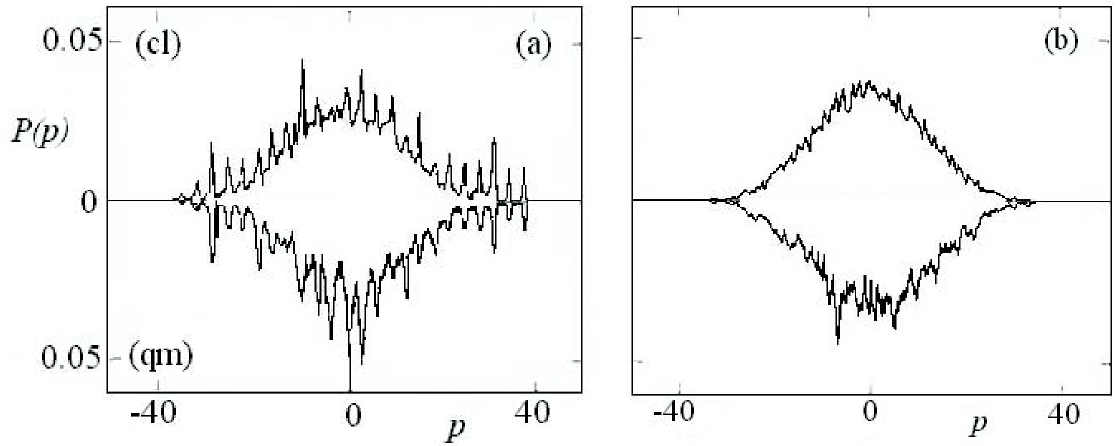






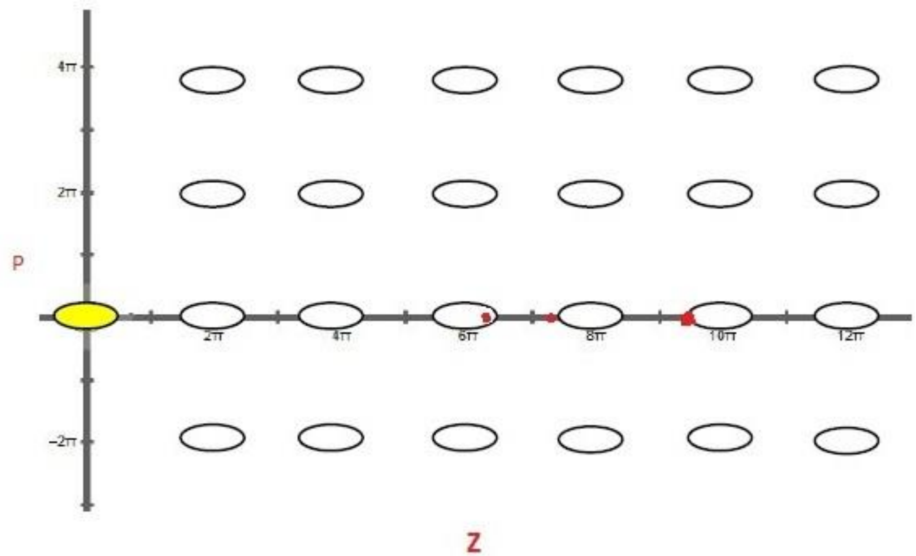
Javed Akram, Khalid Naseer , Inam-ur Rehman and Farhan Saif, Mathematical Problems in Engineering , Volume 2009

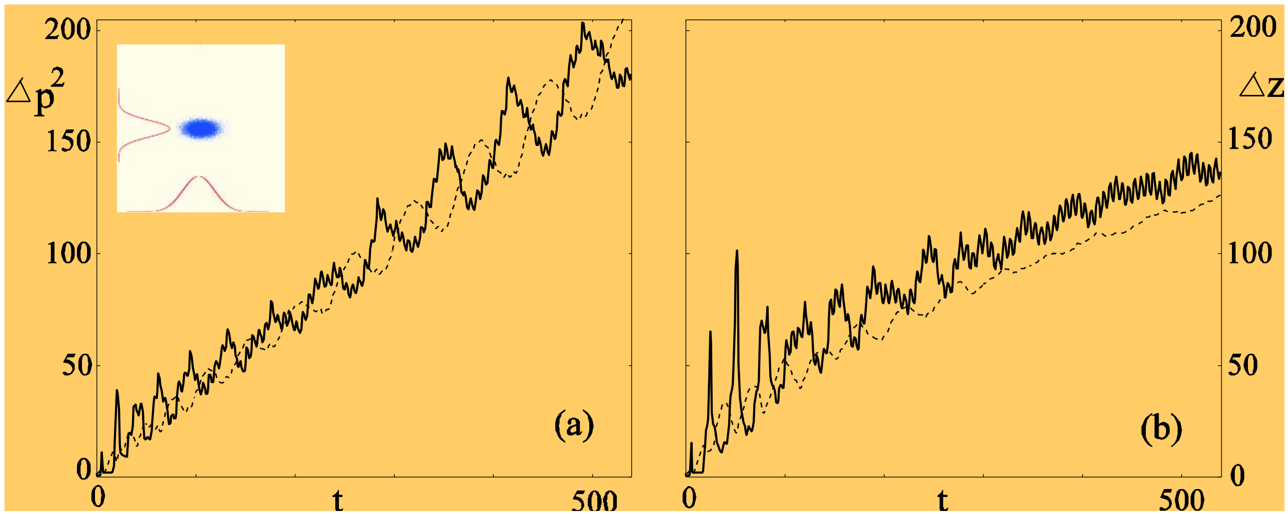
Probability Distribution



$\lambda = 1.71$ $\lambda = 2.4$

$$P(p) = N \exp\left(-\frac{p^2}{4\Delta p^2}\right) \sum_{n=-\infty}^{\infty} \exp\left\{-\frac{(p-n\pi)^2}{4\varepsilon^2}\right\}$$


4π
2π
P
2π
4π
6π
8π
10π
12π
−2π
Z



$\lambda = 1.71$

$\lambda = 2.4$

$$\Delta p^2 \propto t$$

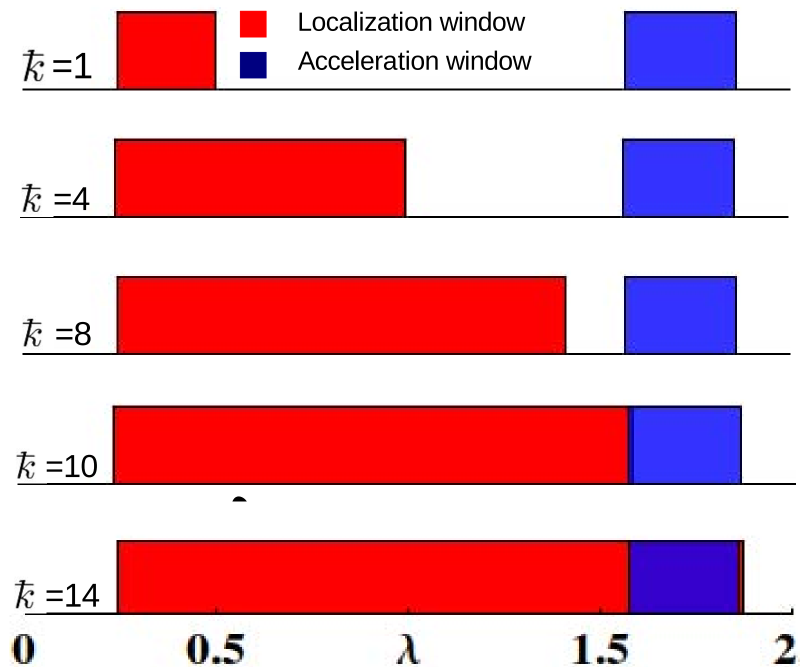
$$\Delta p^2 \propto t^x \quad \text{where} \quad x < 1$$

F. Saif, I. Rehman, Phys. Rev. A. 75, 043610 (2007).

Overlap of acceleration and localization regimes

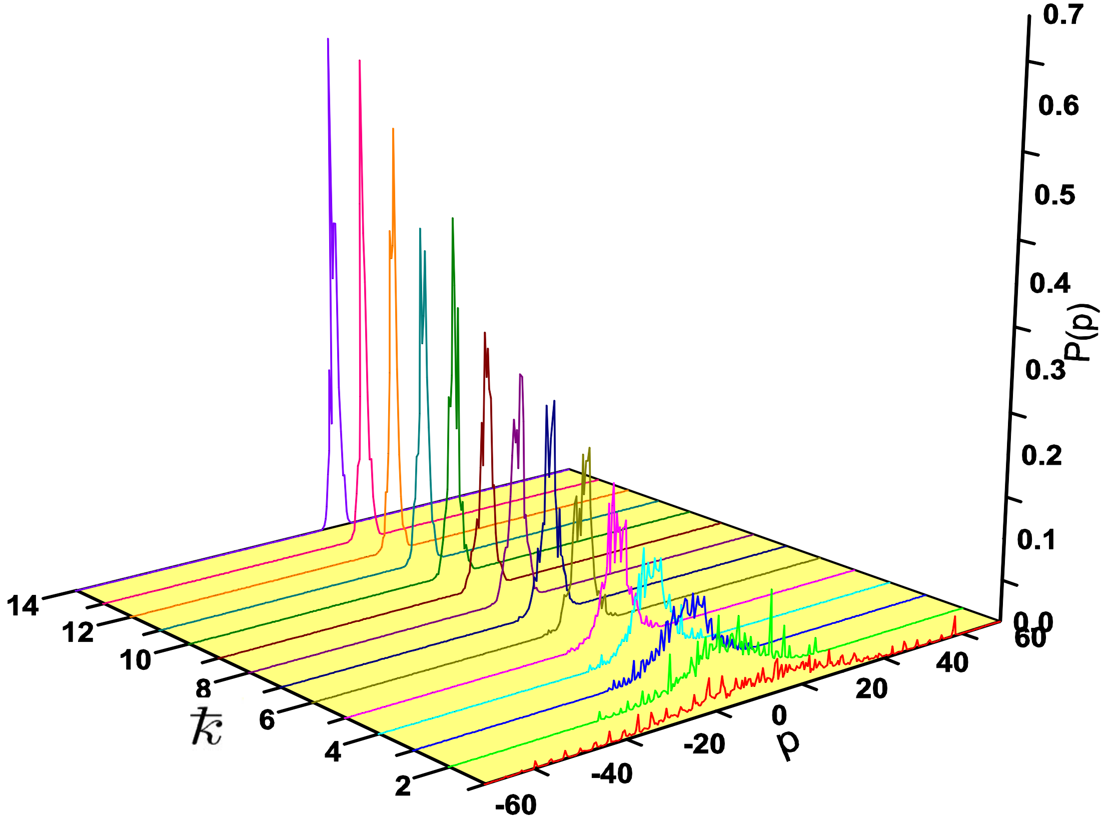
$$0.24 < \lambda < \frac{\sqrt{ƛ}}{2}$$

$$s\pi \le \lambda < \sqrt{1+(s\pi)^2}$$



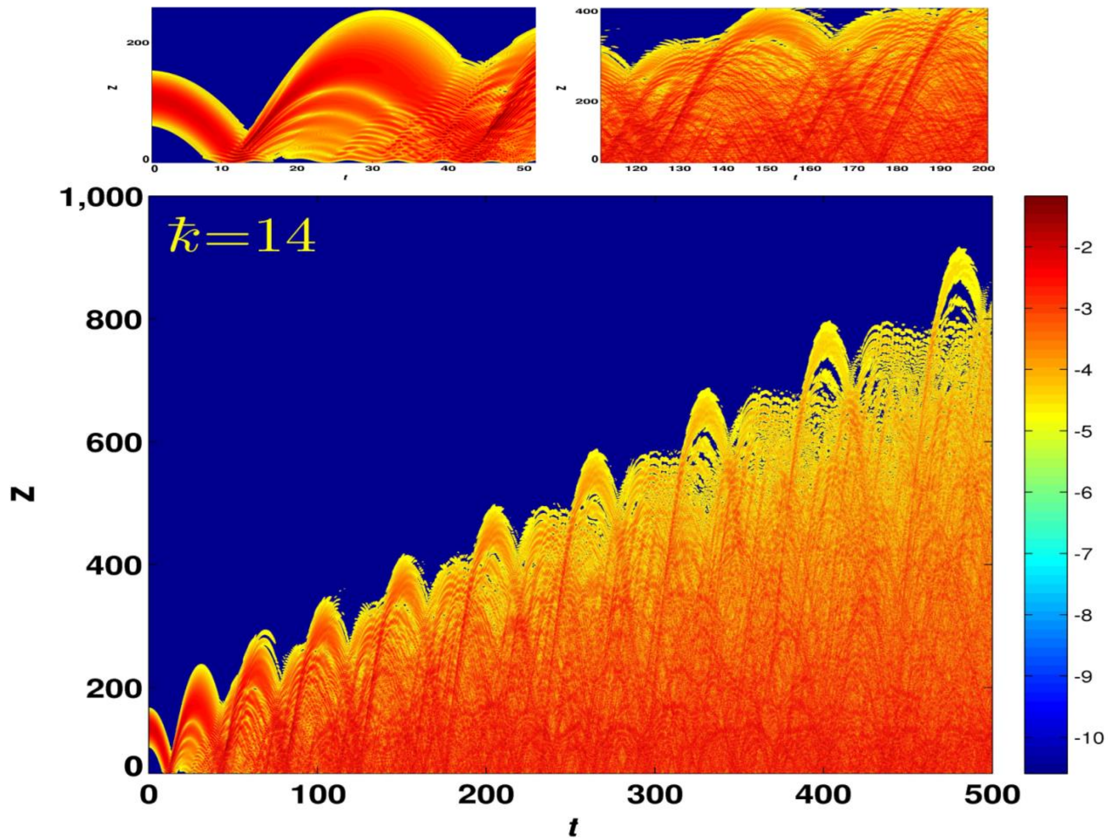
ƛ = ƙ

Probabilty Distribution



Variance for larger effective Plank constant

Probability Distribution in space and time



Thanks!