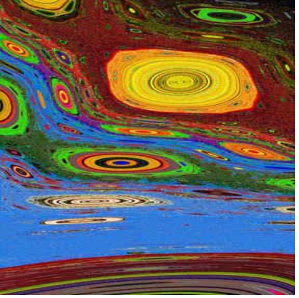


Complex System Physics
May 20 - 24, 2013, Samarkand, Uzbekistan

Viscoelastic behavior, and work and heat-like terms characterizing evolving (time series) systems

(Thermodynamics of relation based systems)

Güngör Gündüz
Orta Doğu Teknik Üniversitesi
(Middle East Technical University, Ankara, Turkey)

(e-mail: ggunduz@metu.edu)

CONTENTS

- **Brief information about the systems considered.**
- **General dynamics of growing systems.**
- **Nonlinearity and scaling relations in growing (Social&Stock market) systems.**
- **Scattering diagrams, and patterns.**
- **Evaluation of pattern shape.**
- **Viscoelastic behavior.**
- **Characterization of time series systems, namely, stock market indices and some social events in terms of work-like and heat-like terms.**

- √ **Ibn-i Khaldoun (1332–1406): Societies are like living organisms; they are born, they grow, and die.**

- √ **Aristotle: Everything in nature is subject to continuous GENERATION and DESTRUCTION (corruption) (<u>everything is transient</u>.)**

- √ **Ancient Greek Natural Philosophy :**
  - **The NATURE is a living organism.**
  - **Everything in nature multiplies itself.**

- √ **Living organisms feed on others, multiply, and die. The growth pattern follows the rules of nonlinear dynamics. The nonlinearity of a system originates from its multiplication at the expense of others.**
  (Chotic growth!)

- √ **Systems are autocatalytic.**

Self multiplying (autocatalytic) systems

- All organisms

food + animal → animal + baby animal

- money (in saving acct.) → money + interest (profit)

- burning tree → burning forest

-small gossip → spreading gossip (like epidemic disease)

Uncontrolled (explosive or chain reaction) Systems

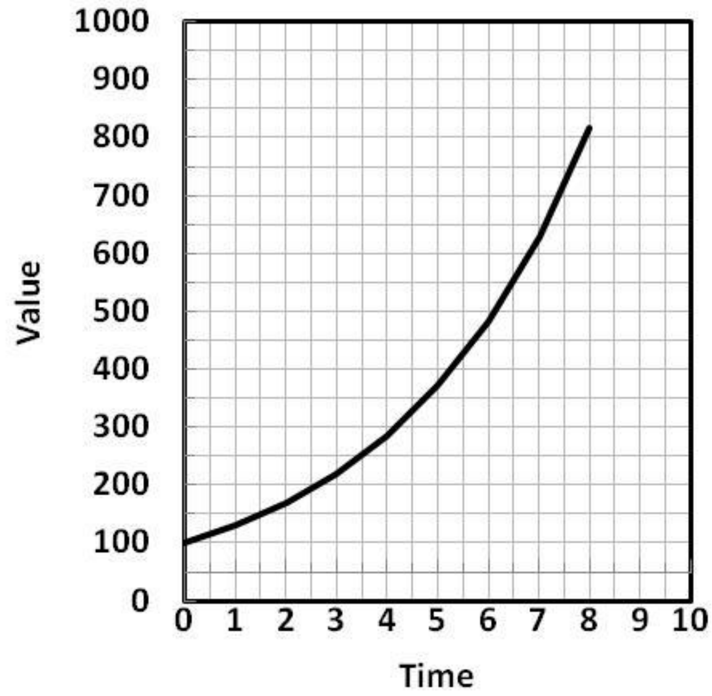
$$N_1 = N_0(1+i)$$

$$N_2 = N_1(1+i) = N_0(1+i)^2$$

.

.

$$N_n = N_0(1+i)^n$$



$i =$ interest for saving account

$$U^{235} + n \rightarrow X + Y + \gamma + 2.43n$$

$i =$ # of neutrons in fission $= 2.43$

Self Controlled Systems (Bacterial growth on an agar)

$$N_{n+1} = N_n(1+i)$$

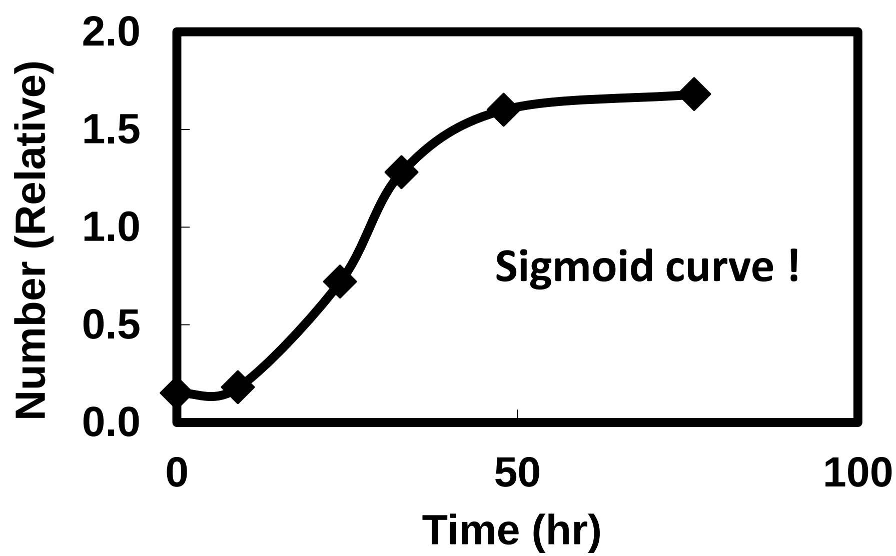
$$i = \frac{N_{n+1} - N_n}{N_n}$$

Verhulst: 'i' varies with population size, and $i \rightarrow N_{lim} - N_n$

Let $N_{lim} \rightarrow 1$; thus, $i = r(1-N_n)$

$$N_{n+1} = N_n(1+r) - rN_n^2$$

$$N_{n+1} = N_n(1+r) - rN_n^2$$



Externally Controlled Systems (Lotka-Volterra problem)

Carrot + Rabbit $\rightarrow\rightarrow$ Rabbit + Rabbit

Rabbit + Fox $\rightarrow\rightarrow$ Fox + Fox

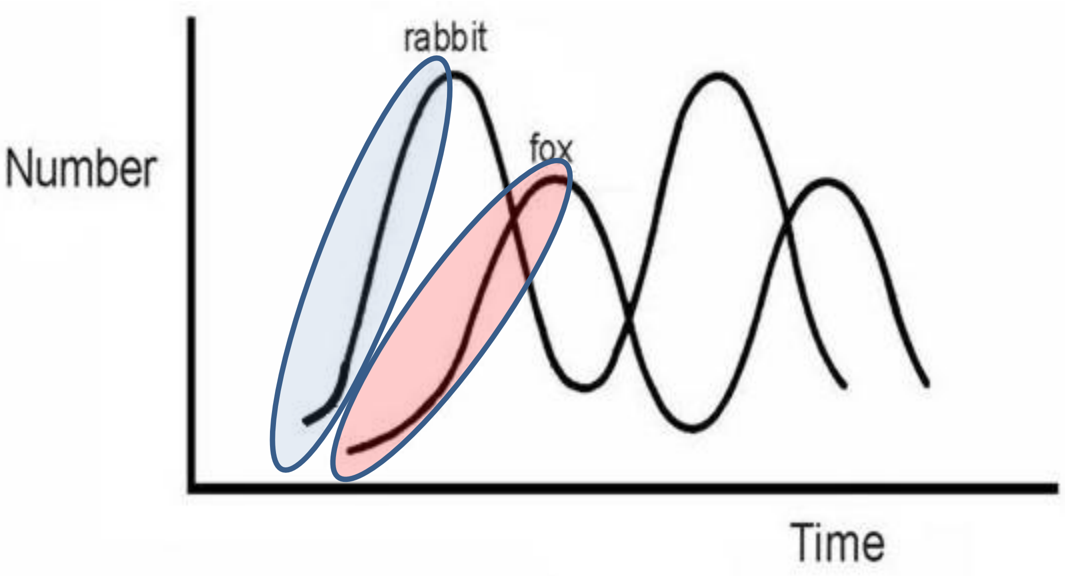
Fox $\rightarrow\rightarrow\rightarrow$ Extinct $\rightarrow$ (Carrots)

$$\frac{d[R]}{dt} = k_1[C][R] - k_2[R][F]$$

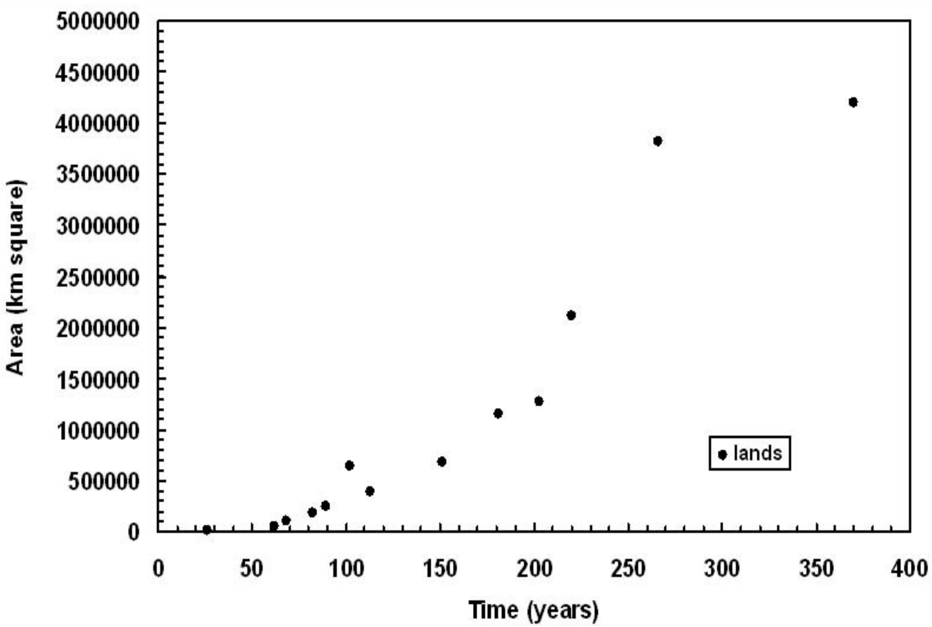
$$\frac{d[F]}{dt} = k_2[R][F] - k_3[F]$$

$$\mathbf{x_{n+1} = \mu\, x\,(1\text{-}x)}$$

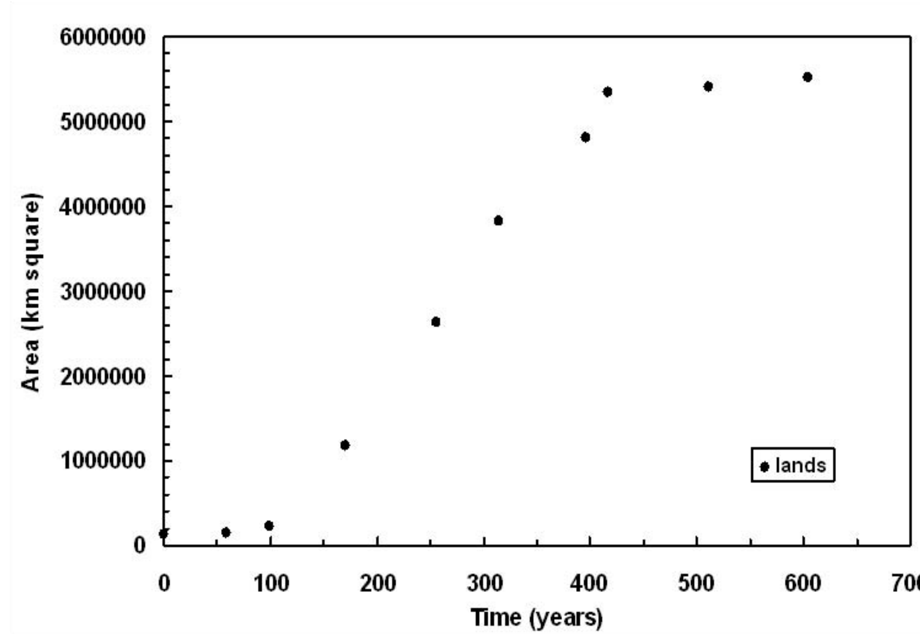
(Logistic equation)

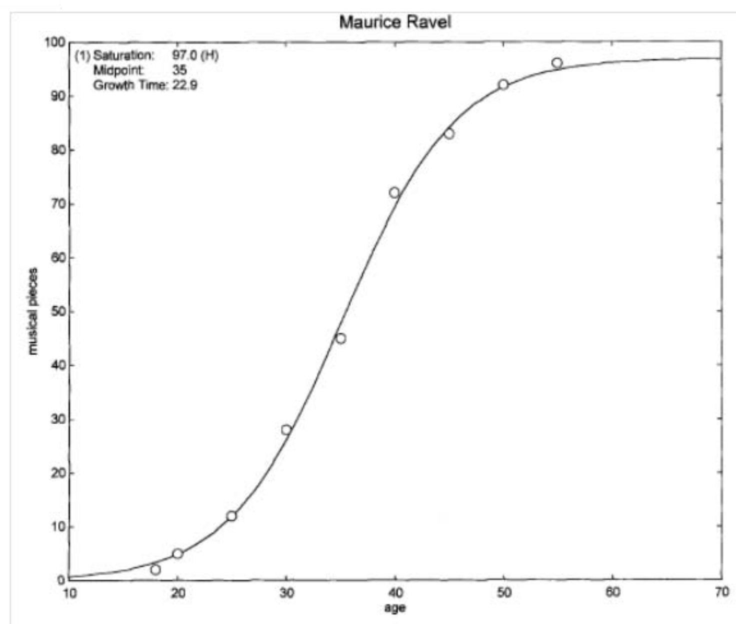


The growth (rise) of Ottoman Empire



The growth (rise) of Roman Empire

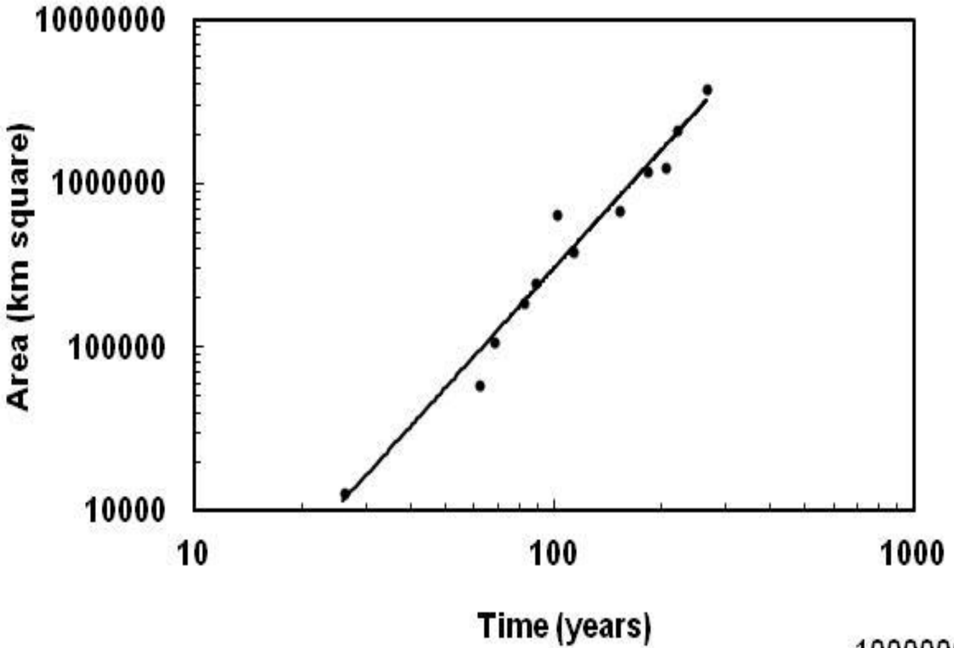




Journal of Mathematical Sociology, 26 (2002) 167-187

C. Marchetti, Productivity vs. Age, 2002

Ottomans

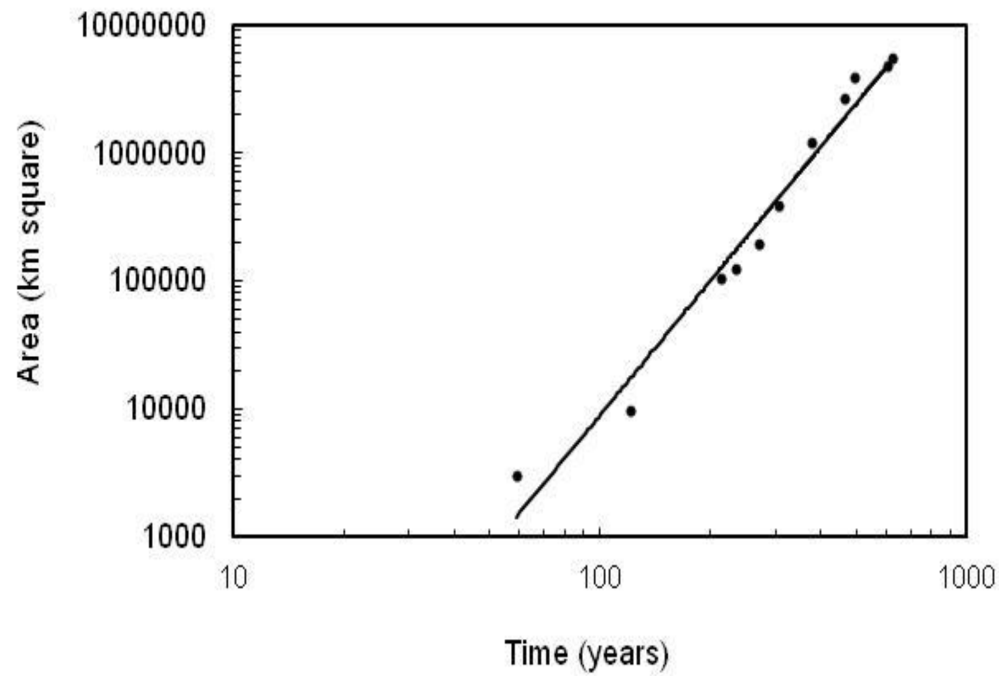


Scaling relation:

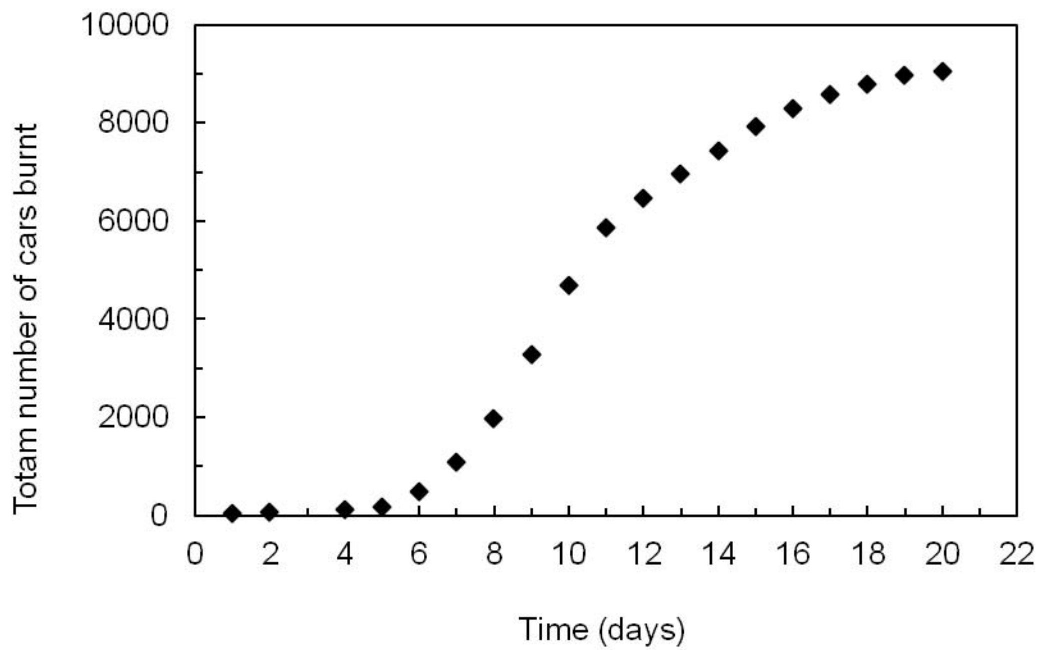
log A = α log t

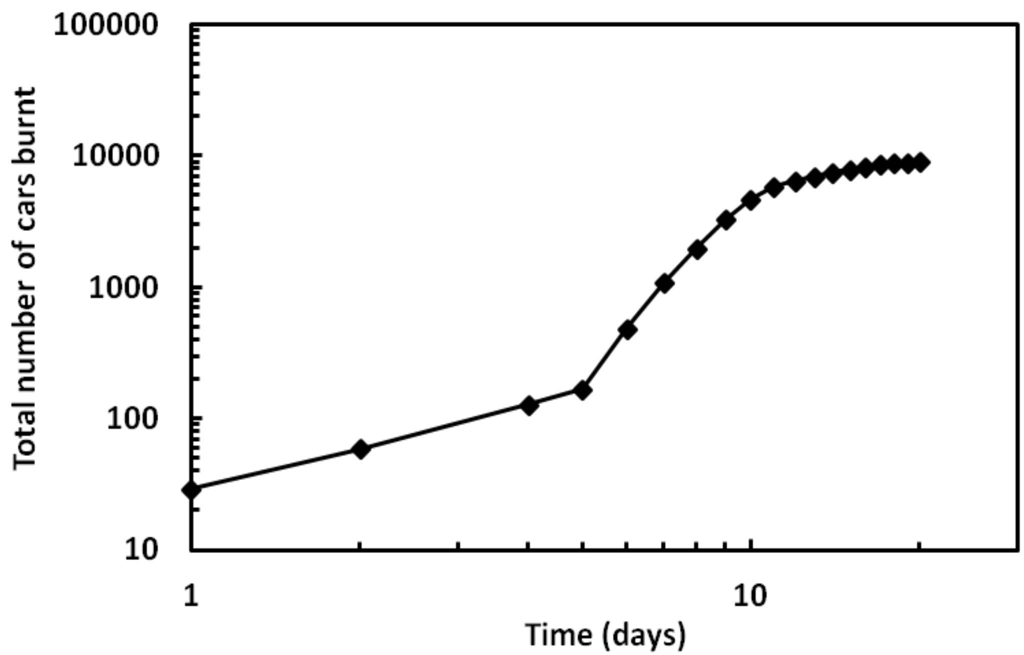
Area ~ t^{α}

Romans

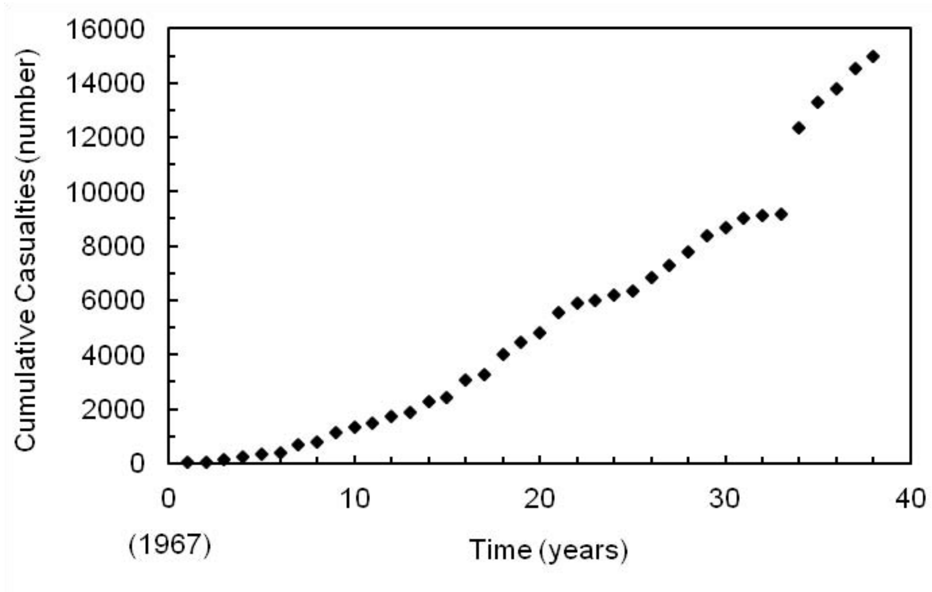


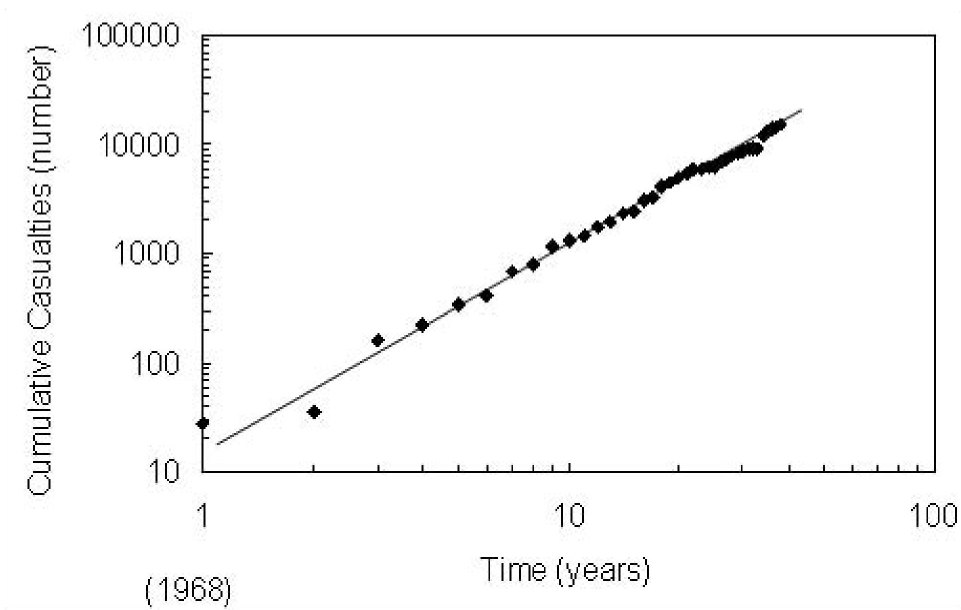
Revolt in France
(December 2005)





International Terror

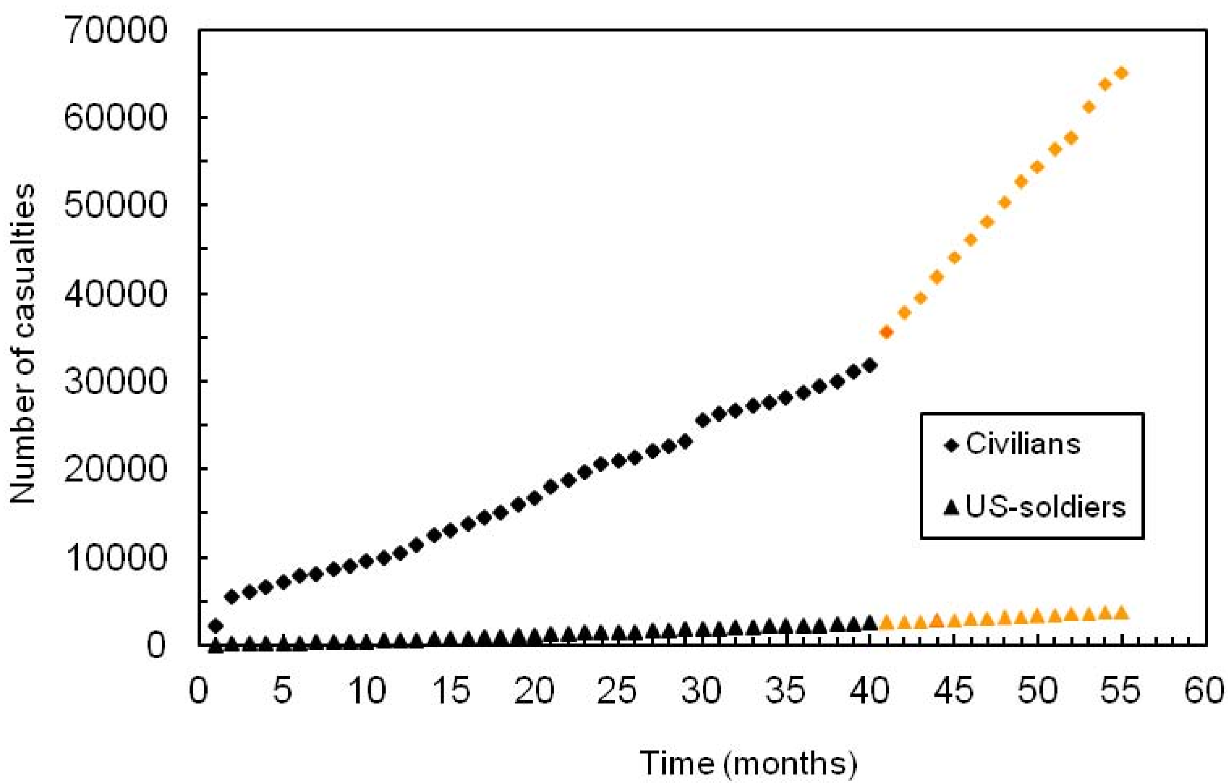




N = 18.6 t$^{1.826}$

Gündüz G., *Physica A*, 376 (2007) 579-595

War in Iraq



$$\frac{dN}{dt} = -k\,N(t)$$

$$\frac{dN}{dt} = -k \quad \Rightarrow \quad \#casualties = N - N_0 = kt$$

Zero order chemical reaction!

Order of reaction ≥ 2

Zero order reaction !

Different Approaches in Literature:

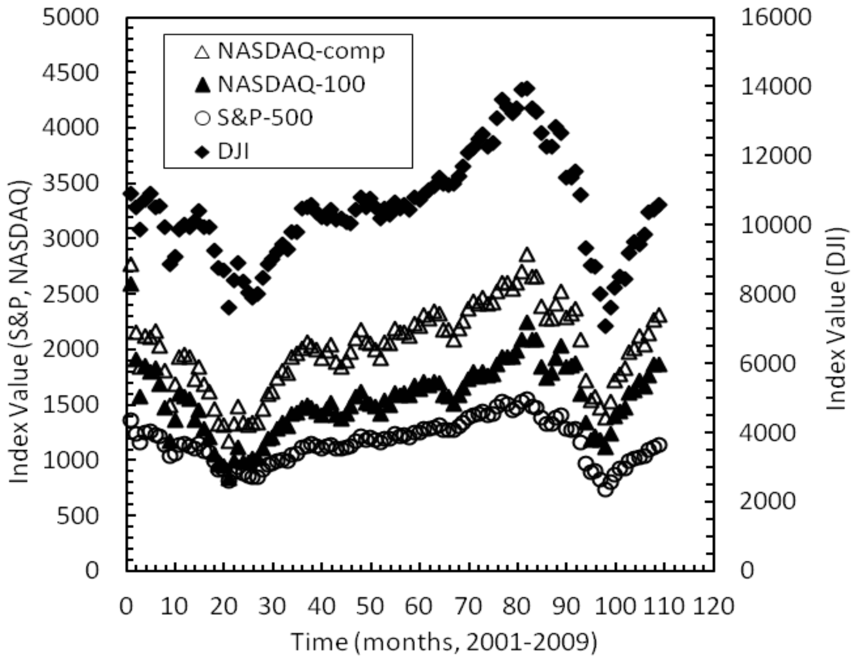
Social Systems

- Galam: meth. of statis. mech. (probabilty, spin sys., percolation)
- Sznajd: Ising spin model,
- Phase transition & percolation models,
- The Fermi equation,
- Chaos & scaling relations.

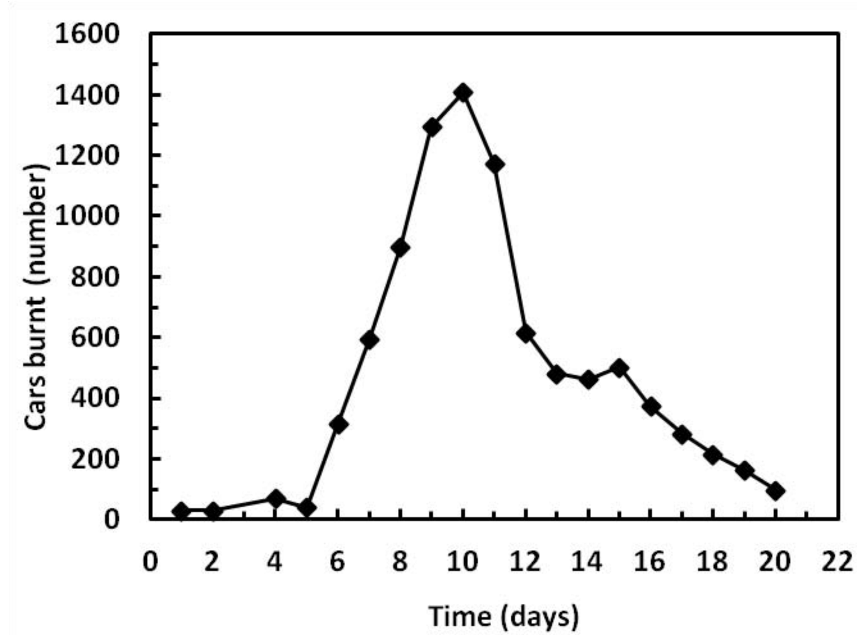
Econophysics

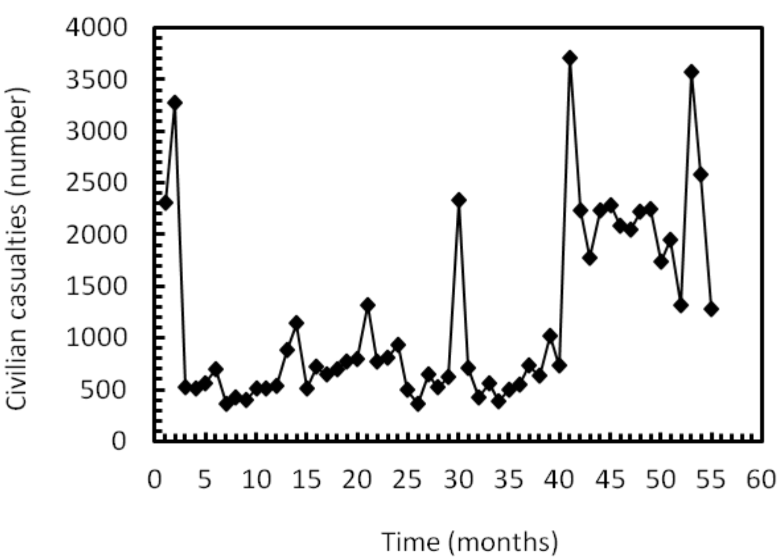
- Pareto law, probabilistic methods, Gibbs distribution, entropy,
- Chaos & scaling relations (Mandelbrot), non-equilibrium dynamics,
- Liouville, Boltzmann, Langevin, Fokker-Planck, Black-Scholes eq.s,
- Spin models, phase plots, phase transition, wavelet techniques,
- Topology, anomalous diffusion, Le Chatelier principle,
- Networks, graph theory,
- Quantum field theory, path integrals, fuzzy logic.

Time Series Systems (Relation Based Systems)

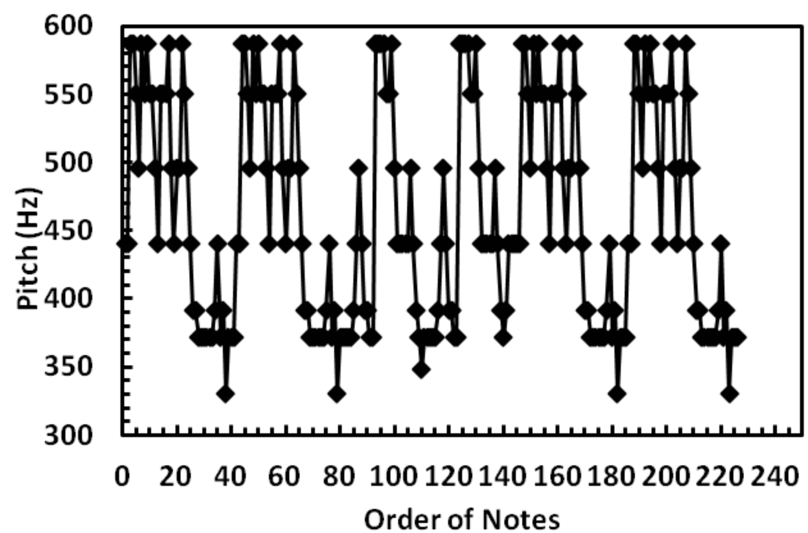


(Revolt in France)





(Iraq)



Folk song: Çemberimde Gül Oya

What **forces** drive

- the stock markets, social struggles, peoples' behavior, etc. ?

Takayasu et al. (2006): There is a potential force in market dynamics; it moves according to its own traces as in Newtonian mechanics.

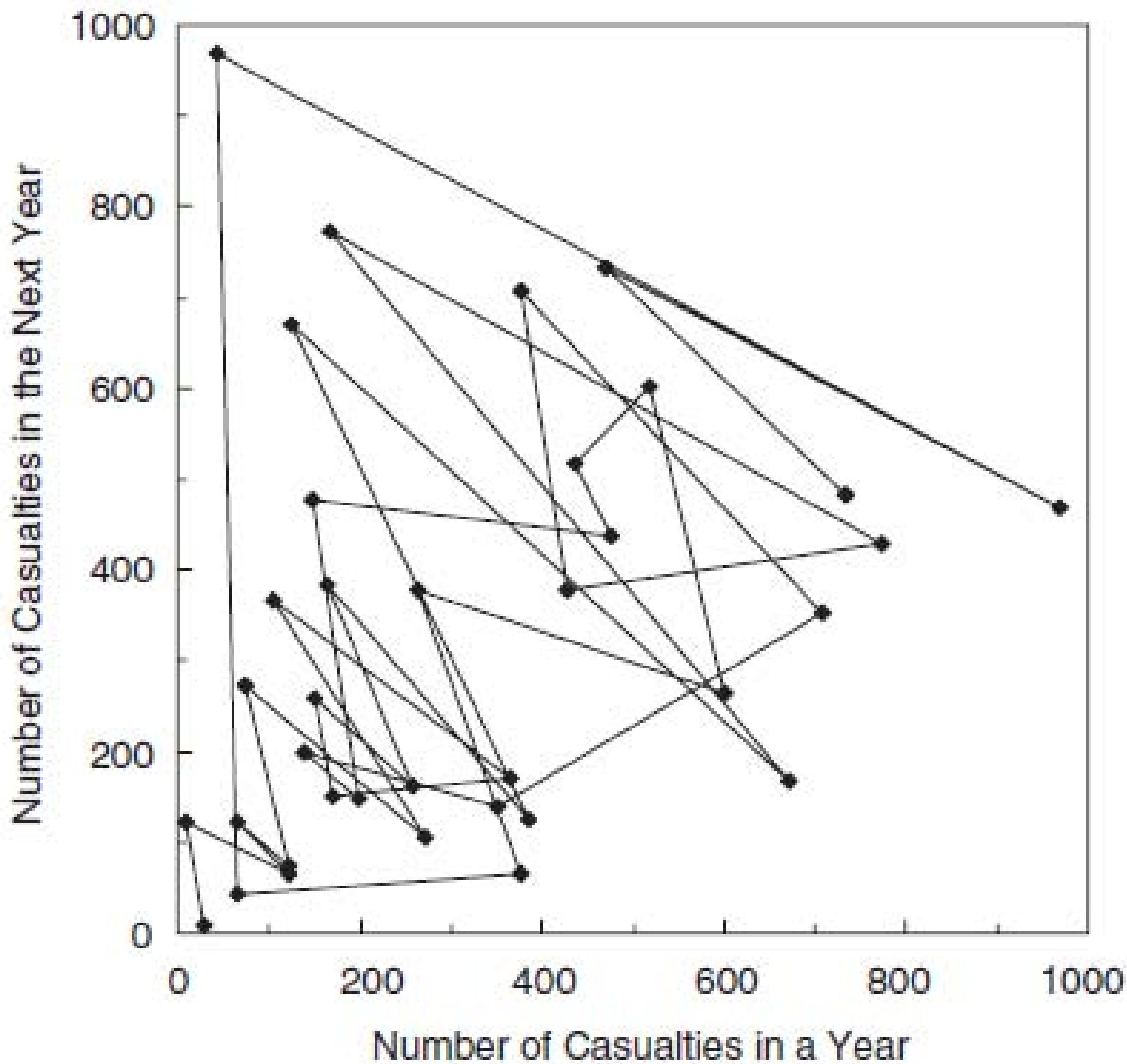
Alfi et al. (2007) : Attractive and repulsive forces affect the index.

Canessa (2009): The stock index dynamics is influenced by a moving average of the index itself, and there can be both attractive and repulsive forces affecting the index like mass-spring system.

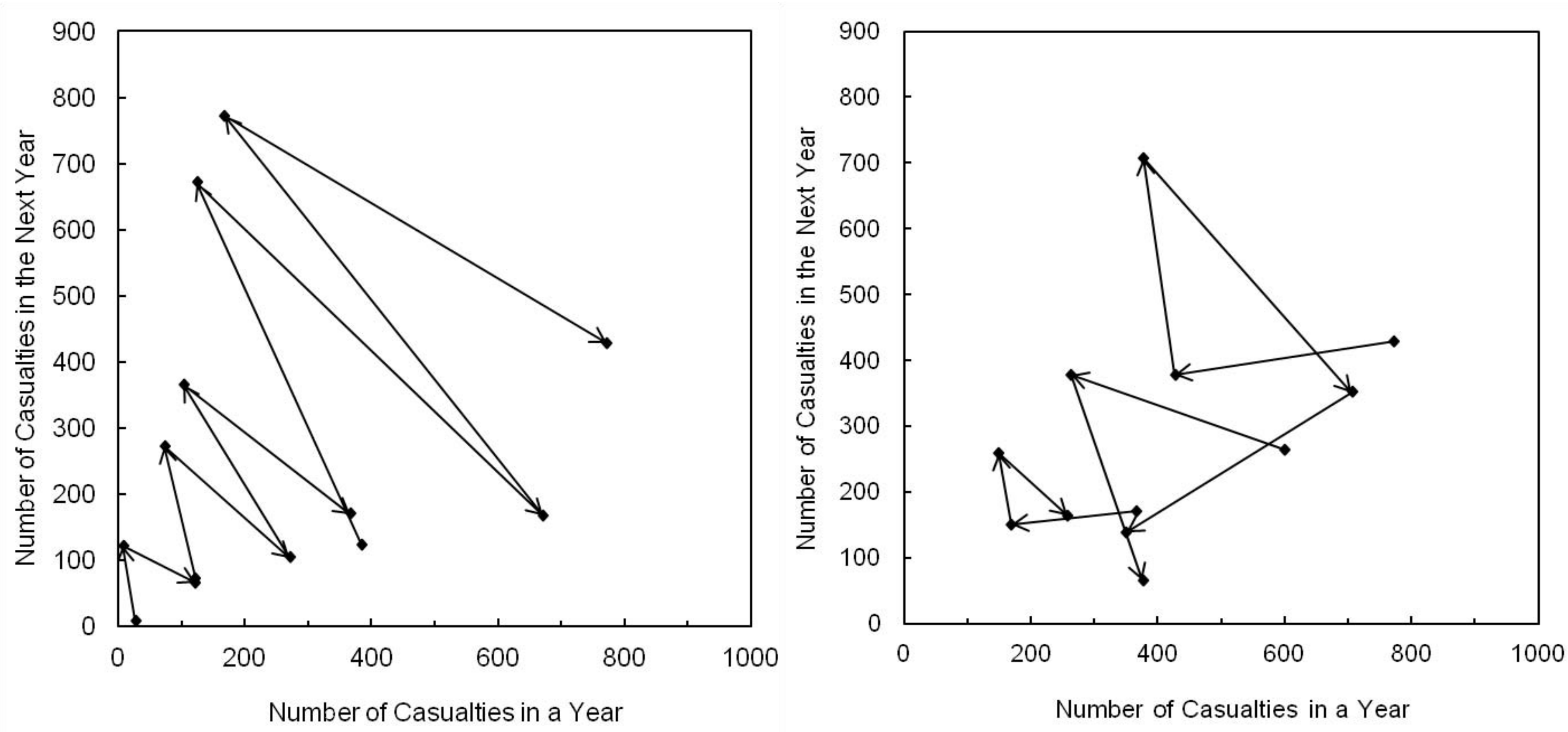
Tuncay (2006): Energy can be attributed to stock prices.

- Aristotle : The present 'actuality' is the 'potential' of the future .
- Today's moving average influences tomorrow's price.
- How do we relate TODAY to TOMORROW?

Scattering Diagram of International Terror

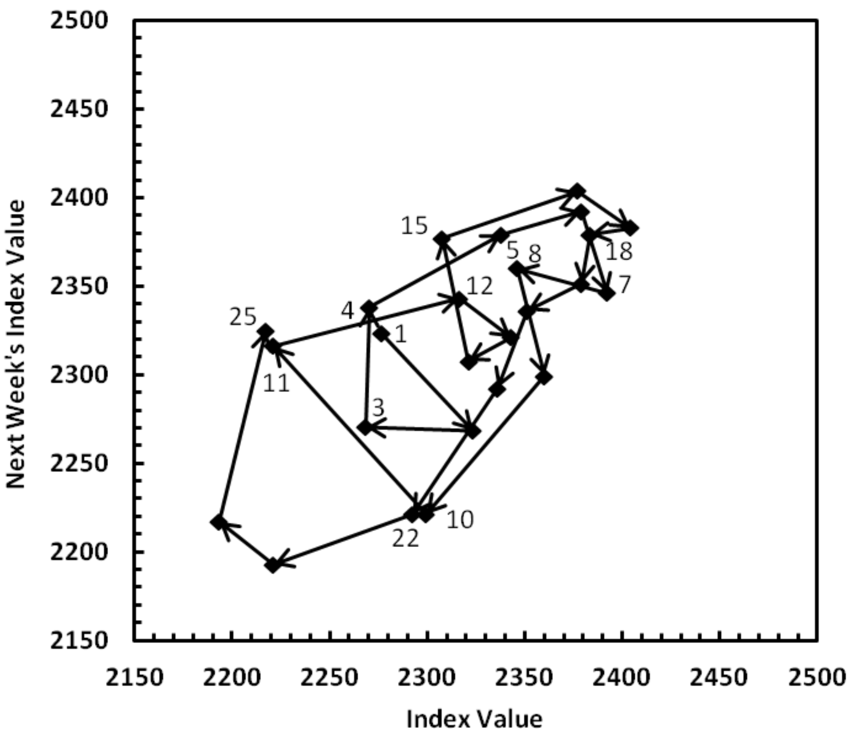


Stretching & Folding back

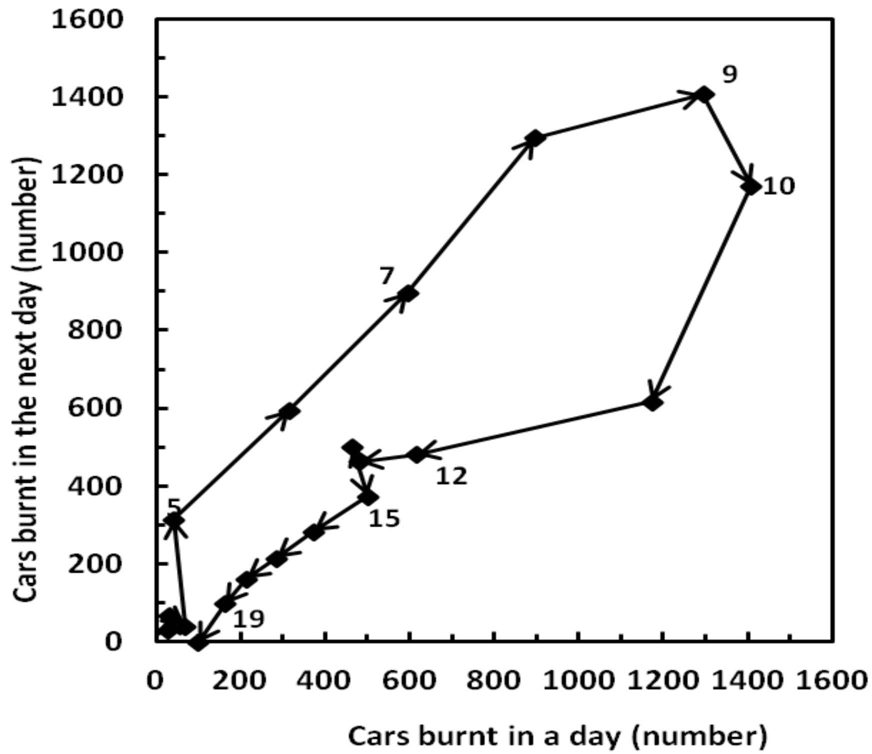


International Terror (some of the paths)

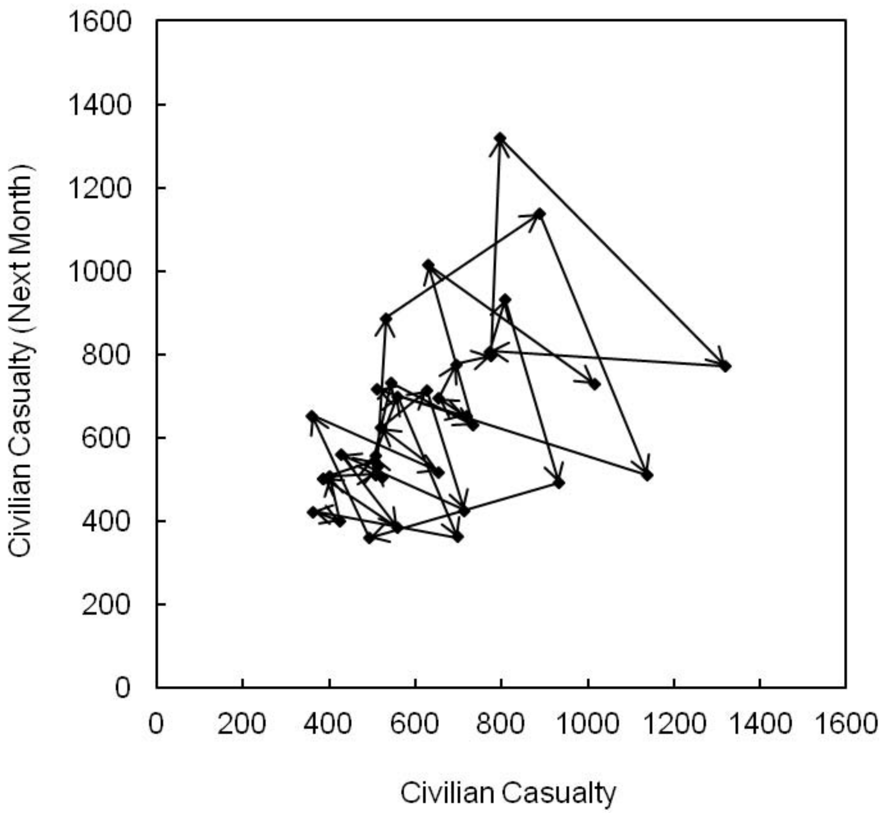
Scattering Diagrams

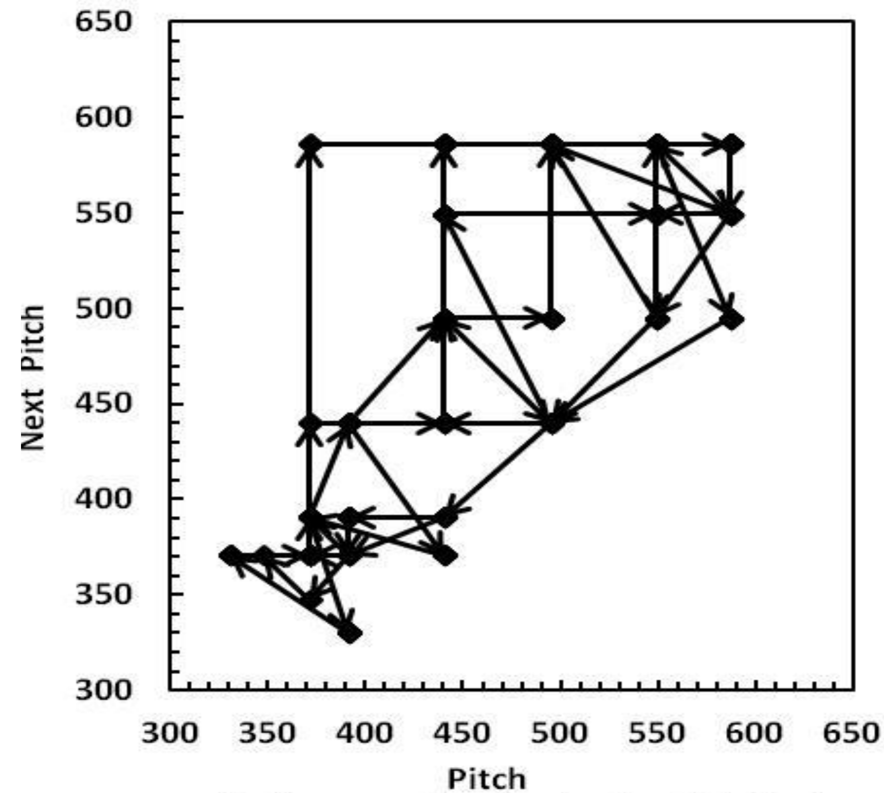


NASDAQ-100 (Jan-June-2011)



(Car Burning Revolt in France)

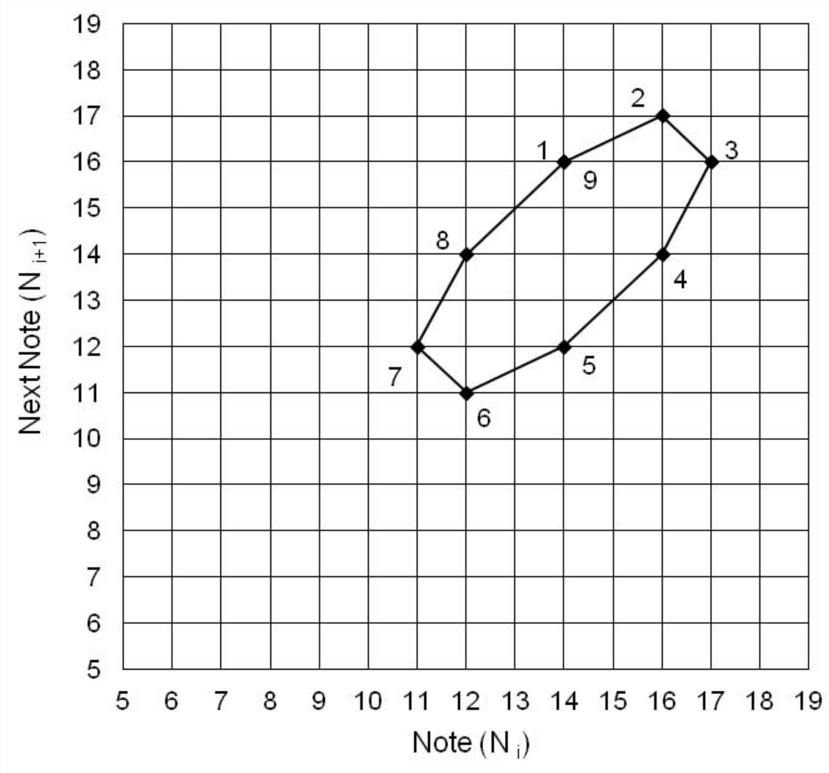


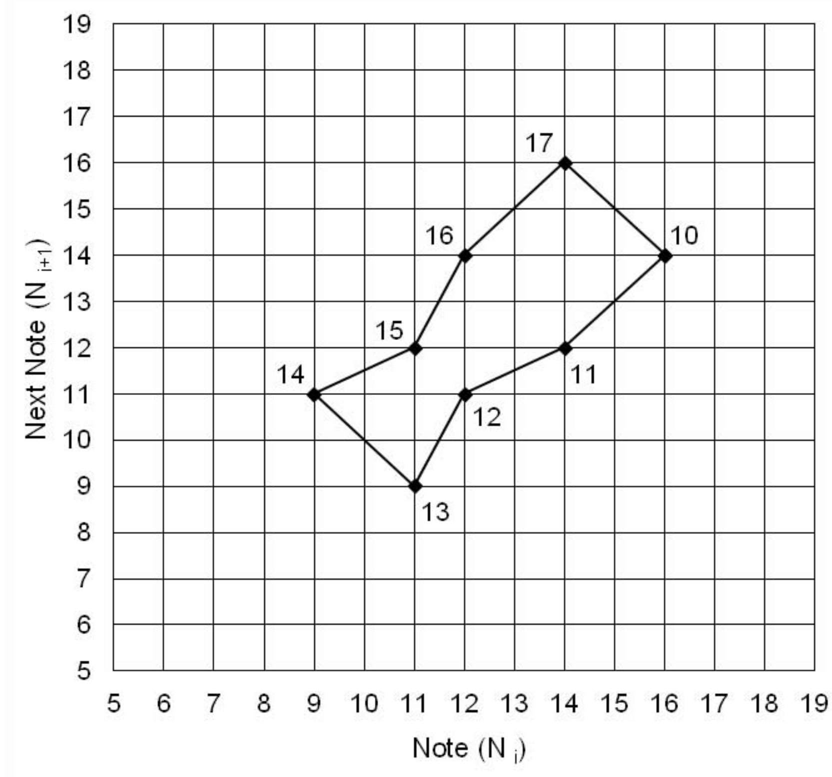


The CURRENT STATE is CHANGED into NEXT STATE.

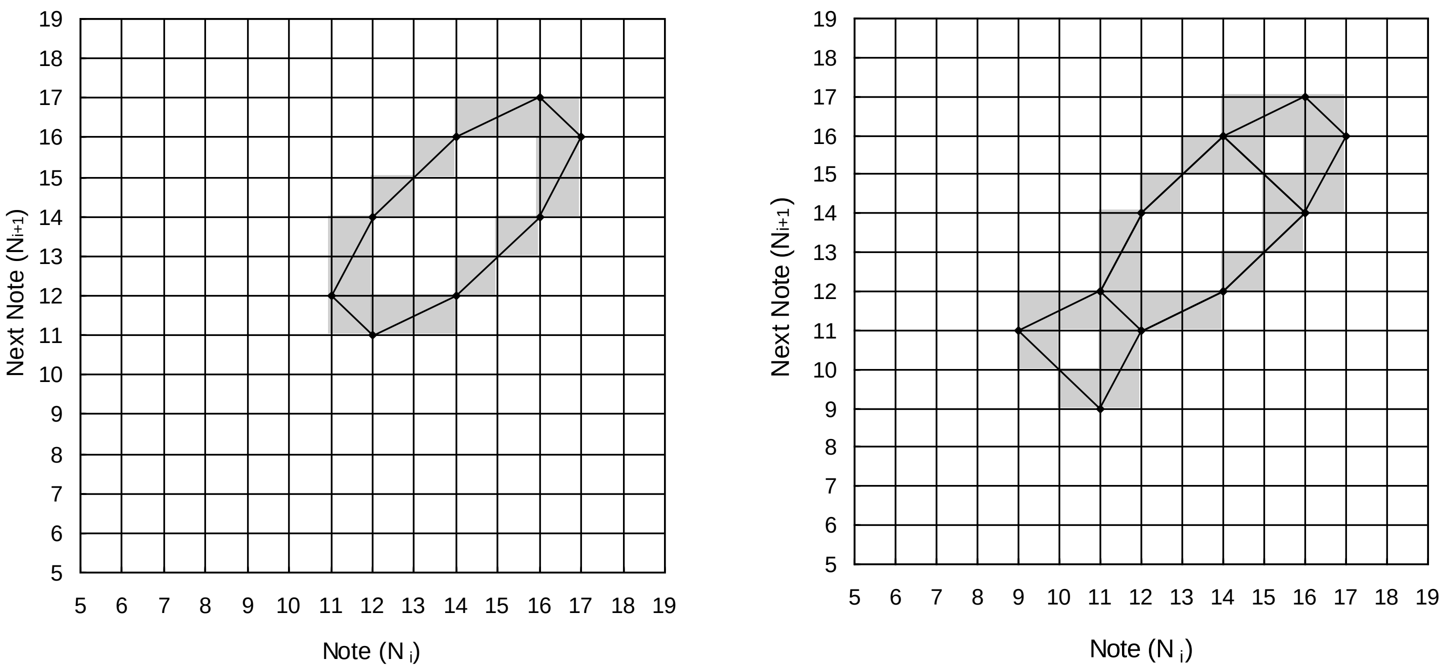
The CURRENT STATE is DEFORMED into NEXT STATE.

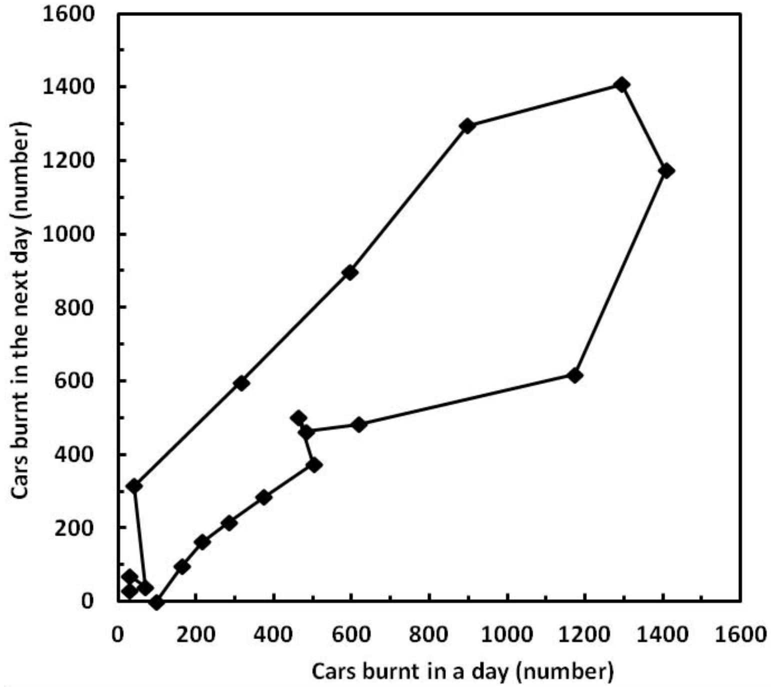
The pattern (or the directed graph) in a scattering diagram is specified by the LENGTHS and their SLOPES.

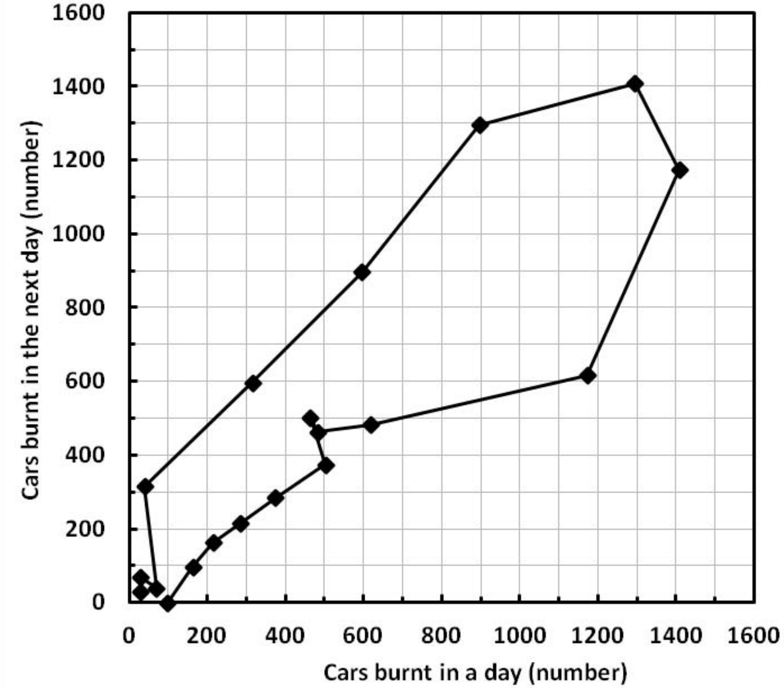


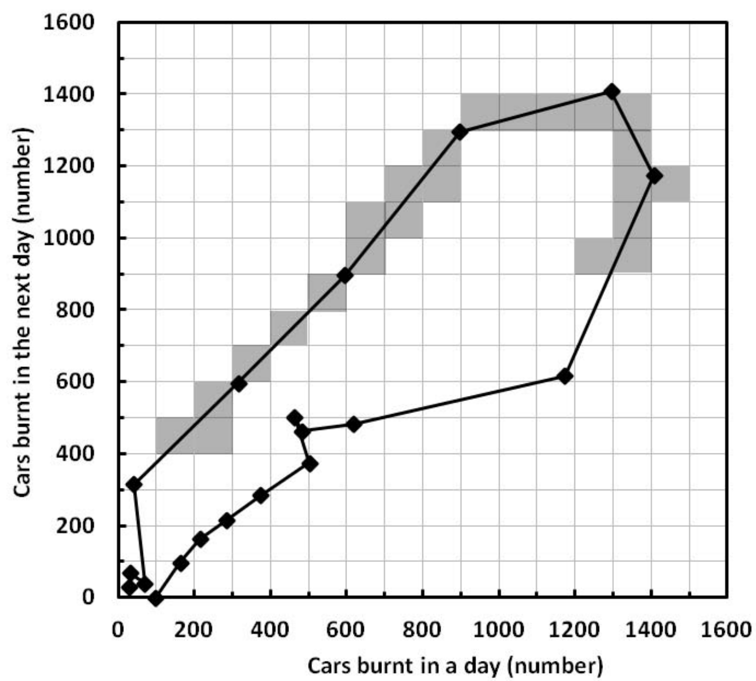


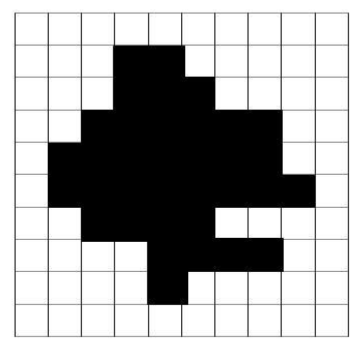
Physica A 357 (2005) 565–592


Next Note (Ni+1)
Note (N i)
Next Note (Ni+1)
Note (N i)







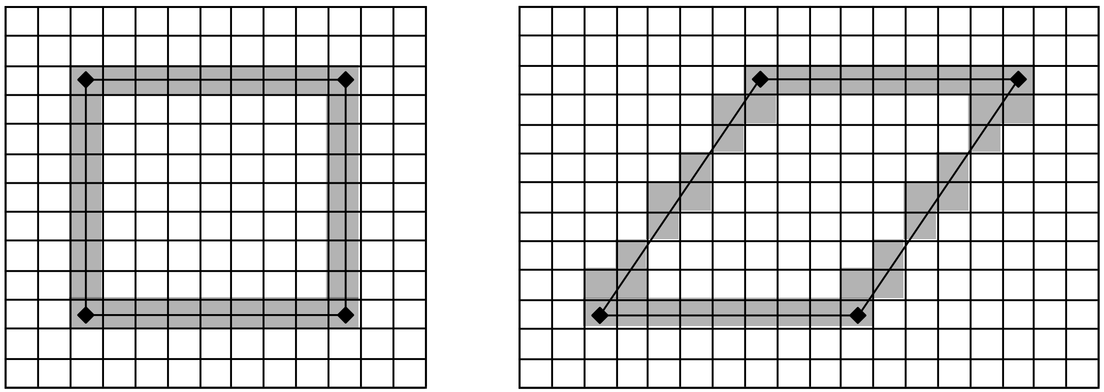


Animal diagram
(Lattice animal)

NETWORK PATTERN AS A DEFORMED SURFACE

(AN ANIMAL DIAGRAM (OR LATTICE ANIMAL) AS A DEFORMED SURFACE)

Deformation of square



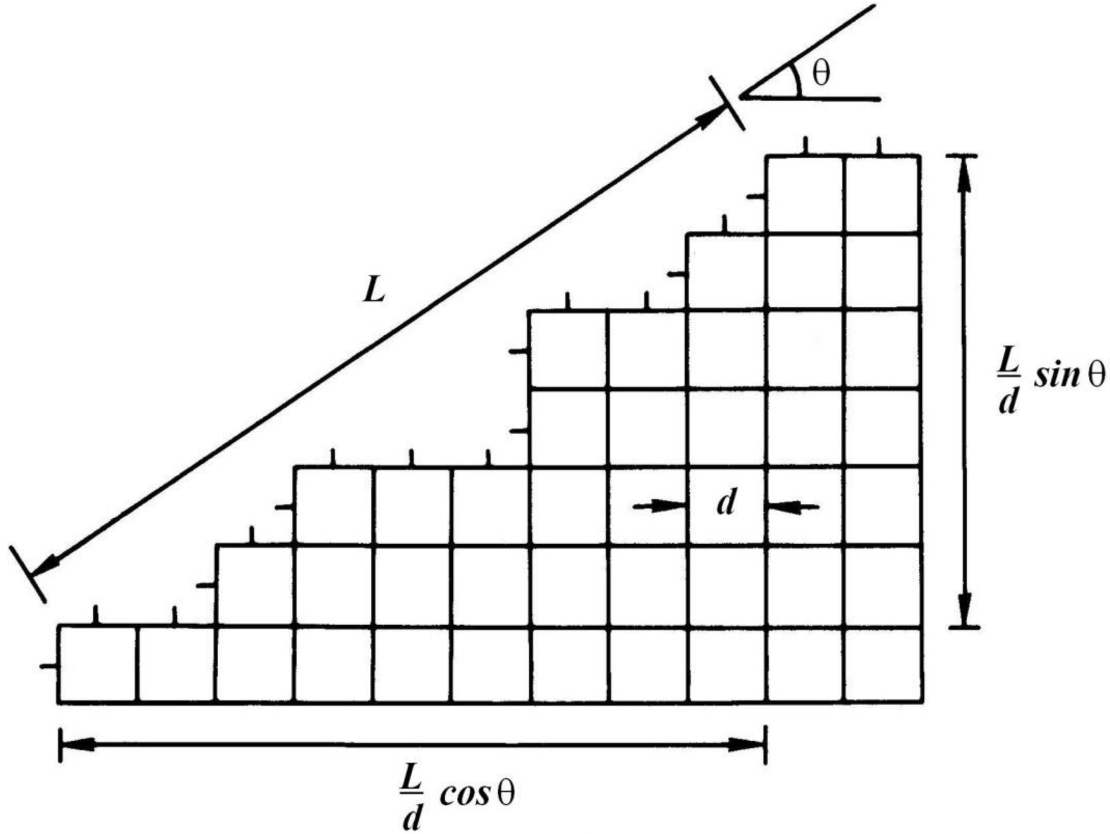
$$dU = dq - dW + \sigma\, dA$$

Only surface changes !

$$dS = -\sigma \frac{dA}{dT}$$

Surface of a crystalline material

(*representative of an animal diagram, or lattice animal*)



$$\#\ of\ broken\ bonds\ along\ x\text{-}direction = \frac{L}{d}\cos\theta$$

$$\#\ of\ broken\ bonds\ along\ y\text{-}direction = \frac{L}{d}\sin|\theta|$$

$$total\ \#\ of\ broken\ bonds = n_d = \frac{L}{d}(\cos\theta + \sin|\theta|)$$

$$\#\, of\ \ broken\, bonds\, in\, the\ \ perpendicular\ direction = \frac{L}{d}$$

$$total\, \#\, of\ \ broken\, bonds\, on\, the\, entire\, area = \frac{L}{d} \times \frac{L}{d} (cos\,\theta + sin\ |\theta|)$$

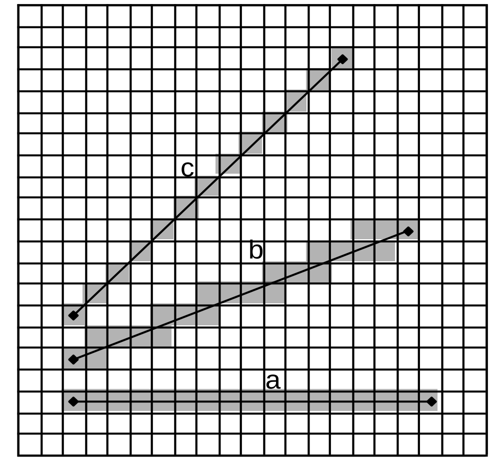
By letting L=1, and attributing ‘$\grave{o}/2$’ energy to each broken bond,
the surface energy E can be given by,

$$E = \left(\frac{\grave{o}}{2} \right) \frac{1}{d^2} (cos\,\theta + sin|\theta|)$$

If only the broken edge is taken into consideration,

$$\sigma_\theta = \frac{\grave{o}}{2d} (cos\,\theta + sin\ |\theta|)$$

$$E=\left(\frac{\text{ò}}{2}\right)\frac{1}{d^{2}}(cos\,\theta+sin\left|\theta\right|)$$



$$\sigma_{\theta}=\frac{\text{ò}}{2d}(cos\,\theta+sin\,\left|\theta\right|)$$

$\sigma_{\theta}=\text{ò}/\sqrt{2}d$ when $\theta=45^{0}$; maximum distortion, most disordered state.

$\sigma_{\theta}=\text{ò}/\,d$ when $\theta=0^{0}$; no distortion, most ordered state.

$$\sigma_{\theta}=\frac{\text{ò}}{2d}(cos\,\theta+sin\,\left|\theta\right|)=\frac{\text{ò}}{\sqrt{2}\,d}cos\,(\theta-\pi/4)$$

An animal (i.e. lattice animal) can be characterized by its number of edges, or by its periphery. Each edge can be simply expressed in terms of one or more integer multiples of the unit length '*d*' of the unit square. Since 'σ_i' denotes the energy of an edge, the '*i*' th edge with length 'ℓ_i' has an energy of '$\sigma_i \ell_i$'.

The total free energy '*G*' is then simply given by,

$$G = \sigma_1 \ell_1 + \sigma_2 \ell_2 + \sigma_3 \ell_3 .... = \sum \sigma_i \ell_i$$

$$\Delta G = G_2 - G_1 = \sum \sigma_i \ell_i - \sum \sigma_j \ell_j = \Delta n_c \grave{o}$$

where Δn_c is the change in the number of corners.

Since each new corner creates a new unit edge length, the change in free energy becomes equal to '$\Delta n_c \grave{o}$'.

MEASURE OF DEFORMATION

Define,

$$\tilde{r} = \frac{\textit{energy of lattice animal}}{\textit{total energy of unit edges}}$$

$$\tilde{r} = \frac{n_{\mathrm{c}}\grave{o}}{(\sum d)\grave{o}} = \frac{n_{\mathrm{c}}}{n_{\mathrm{d}}}$$

where, $n_{\mathrm{d}} = \sum d$. So, $\tilde{r}$-ratio becomes independent of energy, and it becomes only a **'geometric proportion'**.

It is clear that,

$\tilde{r} \to 1$ as $\theta \to 45^0$

$\tilde{r} \to 0$ as $\theta \to 0^0$

DEFORMATION OCCURS IN BOTH HORIZONTAL AND VERTICAL DIRECTIONS !

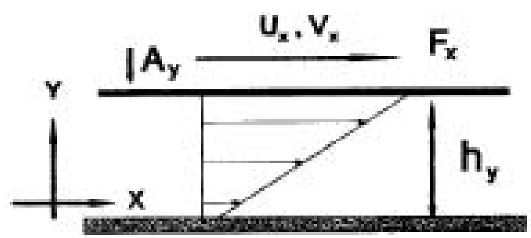
DEFORMATION OF HOOKIAN SOLID AND NEWTONIAN FLUID

For a **Hookian solid** the yx-component of **strain tensor** γ_{yx} is defined in terms of the displacement gradient tensor □$\boldsymbol{u}$ as,

$$\gamma = \nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^{\dagger}$$

Similarly, the **rate-of-strain tensor** for the shearing motion of a **Newtonian fluid** is defined in terms of the velocity gradient $\nabla \boldsymbol{v}$ as,

$$\dot{\gamma} = \nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^{\dagger}$$



Shear associated with rotation generates vorticity, and the vorticity tensor □ω is given by,

$$\omega = \nabla \boldsymbol{v} - (\nabla \boldsymbol{v})^{\dagger}$$

$$\nabla \boldsymbol{v} = \frac{1}{2}(\dot{\boldsymbol{\gamma}} + \boldsymbol{\omega})^{\dagger}$$

That means the velocity gradient tensor $\nabla \boldsymbol{v}$ can be decomposed into its symmetric and antisymmetric parts .

$$\dot{\gamma}_{xy} = \dot{\gamma}_{yx} = \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}$$

$$\dot{\omega}_{xy} = -\dot{\omega}_{yx} = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}$$

So it is clear that,

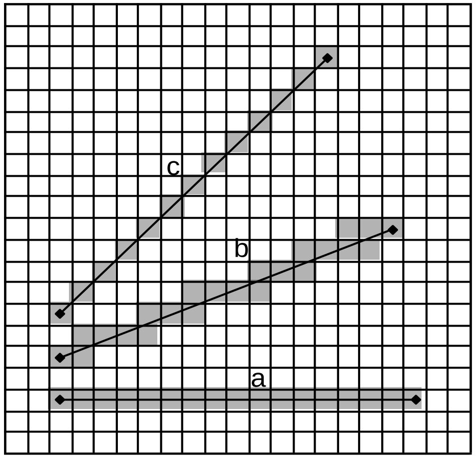
$$\dot{\gamma}_{xx} = 2\frac{\partial v_x}{\partial x} \quad \text{and} \quad \dot{\gamma}_{yy} = 2\frac{\partial v_y}{\partial y}$$

Similarly,

$$\gamma_{xy} = \gamma_{yx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$$

Then, one can easily get,

$$\dot{\gamma}_{xx} = 2\frac{\partial v_x}{\partial x} \quad \text{and} \quad \dot{\gamma}_{yy} = 2\frac{\partial v_y}{\partial y}$$



Similarly,

$$\gamma_{xx} = \gamma_{yy} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \qquad \gamma_{xy} = \gamma_{yx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$$

The edge of a unit square corresponds to the unit displacement of γ_{xx}, (or γ_{yy}) along the x-axis (or y-axis). n_d corresponds to the number of unit edges both along the x- and y-directions. That is, every unit displacement of a row along x- direction (i.e. γ_{xx}) or of a column along y- direction (i.e. γ_{yy}) increases n_d. Therefore the total number of unit lengths (e.g. n_d) along both directions is,

$$n_d = n_{d,x} + n_{d,y}$$

$$n_d = \sum (\gamma_{xx} + \gamma_{yy}) = \sum \left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right)$$

In the unit square $dx=dy$ in magnitude. A one-to-one correspondence between n_d and the distances in the Cartesian coordinate system can be established.

$$u = xy$$

$$n_d = n_{d,x} + n_{d,y} = y + x$$

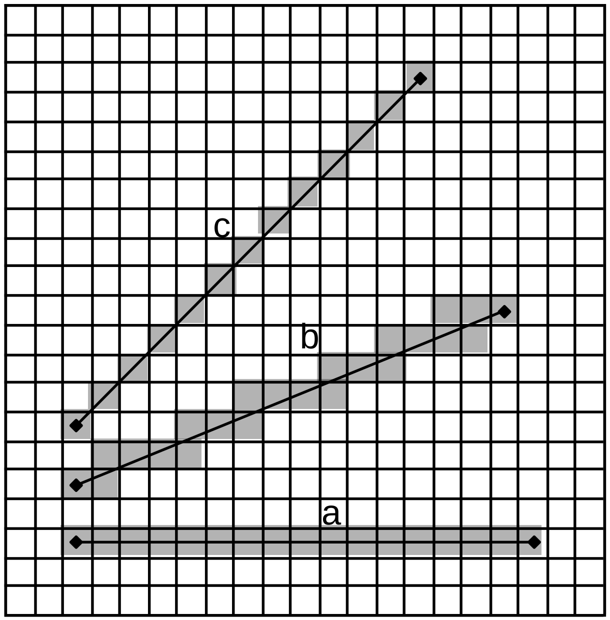
$$dx=dy$$

$$n_c \sim 2\,y$$

If the direction of motion is along the y-direction,

$$n_c \sim 2\,x$$

$$\tilde{r} = \frac{n_\ell}{n_\mathrm{d}} = \frac{n_\mathrm{c}}{n_\mathrm{d}} \sim \frac{2y}{x+y}$$



Consider Figure b;

n_c=14, n_d=23

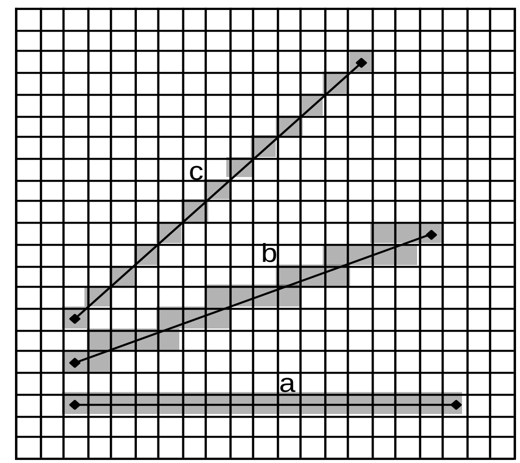
x=16, and y=7

$$\frac{n_\mathrm{c}}{n_\mathrm{d}}\left(=\frac{14}{23}\right) = \frac{2y}{x+y}\left(=\frac{2\times 7}{16+7}\right)$$

Now we can define deformation in a different way,

$$\tilde{r} = \frac{energy\ of\ lattice\ animal}{total\ energy\ of\ unit\ edges} = \frac{n_c\,\text{ò}}{(\sum d)\,\text{ò}} = \frac{n_c}{n_d}$$

$$\tilde{r} = \frac{\gamma_{xx}}{(\gamma_{xx} + \gamma_{yy})/2}$$



$$\tilde{r} = 2\frac{\gamma_{xx}}{\gamma_{xx} + \gamma_{yy}} = 2\frac{du/dx}{du/dx + du/dy}$$

If we let $u \rightarrow y$,

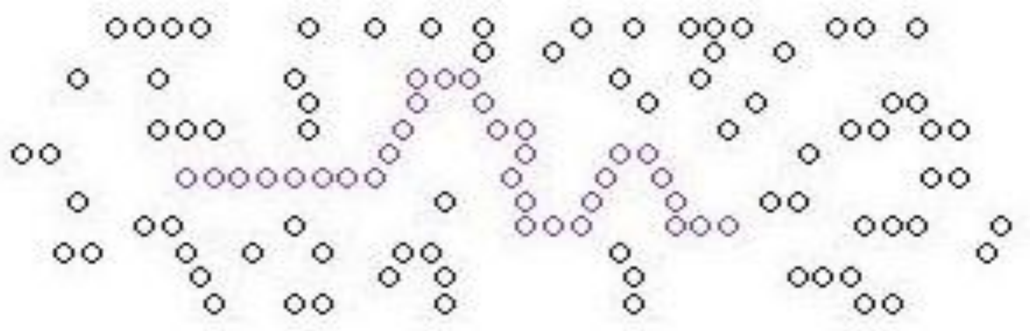
$$\tilde{r} = 2\frac{(dy/dx)}{(dy/dx)+1} = 2\frac{m}{m+1} = 2\frac{tan\,\theta}{1+tan\,\theta} = 2\frac{sin\,\theta}{cos\,\theta + sin\,\theta}$$

GROWING SYTEMS

- In a stock market if a certain stock becomes attractive for some reason and people make up their minds to buy it, it leads to an increase in its market value. That is, it increases its value by picking up contributions from the stock market sea.

- Waves (in plasma): A growing wave picks up fluctuating waves in turbulent plasma.

- A polymer chain grows up by eating up small chains or monomers.



$$Fractional\ change\ in\ polymer\ length = u = dL / L$$

$$\Delta u = \log(L_{i+1} / L_i)$$

$$Financial\ market;\quad log\text{-}return = R = \log(S_{t+\tau} / S_t)$$

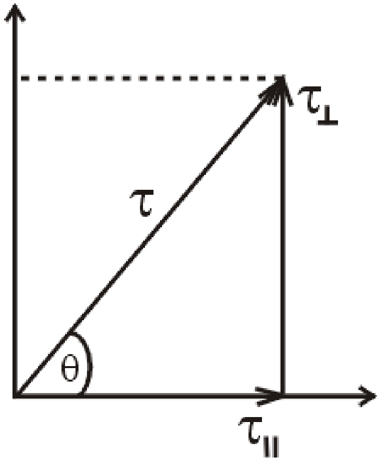
$$S = market\ price\ or\ money\text{-}exchange\ rate.$$

$$overall\ subsequent\ change = U = L_{i+1} / L_i \sim S_{i+1} / S_i$$

$$extent\ of\ \ fractional\ change = V_{i+1} / V_i, \quad V = L,\ S,\ n,\ etc.$$

$$V_i = G\, V_{i+1}$$

$$\tau = G\,\gamma$$



$$\tau^* = \tau_{||} + i\tau_\perp$$

$$\tau_{\square} = G'\gamma, \quad \tau_\perp = G''\gamma$$

$$G = G'\gamma + iG''\gamma$$

$$G' = storage\ modulus \quad (in\text{-}phase)$$

$$G'' = loss\ \ modulus \quad (out\text{-}of\text{-}phase)$$

$$\tilde{r} \sim 2\frac{\sin\theta}{\cos\theta + \sin\theta} = 2\frac{G''}{G' + G''}$$

$$\tan\theta = \frac{G''}{G'} \sim \frac{\tilde{r}}{2 - \tilde{r}}$$

Averaging over length must be done to determine the value of $\tilde{r}$ in an evolving or already evolved network.

$$\tilde{r} = \frac{\sum_{i}^{n} L_i \tilde{r}_i}{\sum_{i}^{n} L_i}$$

$$\frac{G''}{G'} \sim \frac{\tilde{r}_t}{2 - \tilde{r}_t}$$

$$tan\,\theta = \frac{sin\,\theta}{cos\,\theta} = \frac{\tau''}{\tau'} = \frac{G''}{G'} = \frac{\eta'}{\eta''}$$

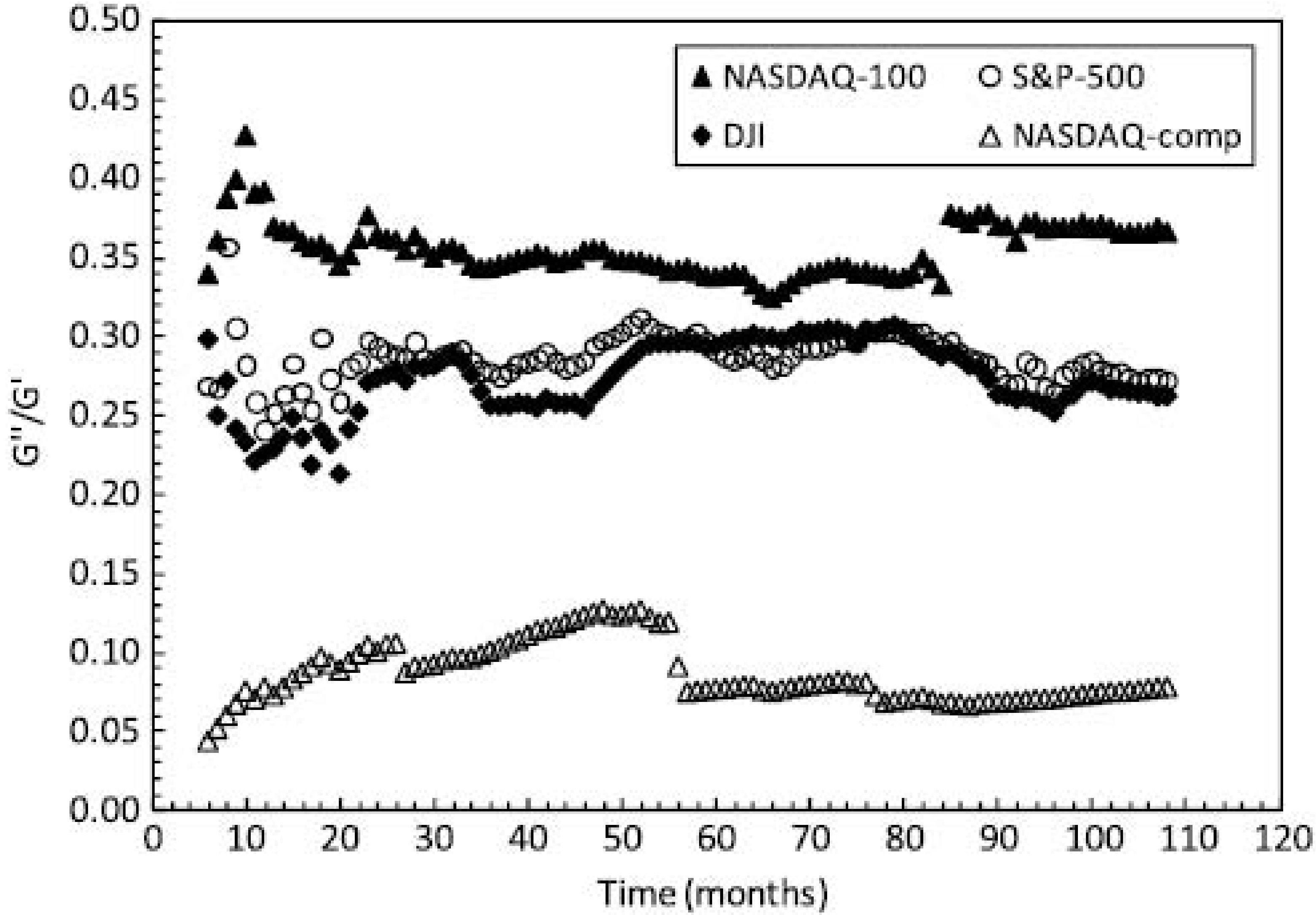
where η' and η'' are the in-phase and the out-of-phase components of the viscosity term.

$$tan\,\theta = \frac{\tau_{\perp}}{\tau_{II}} = \frac{G''}{G'} \sim \frac{\tilde{r}}{2-\tilde{r}}$$

The term '$tan\,\theta$' is called '**loss tangent**'. It denotes the ratio of the amount of energy distributed along the vertical and horizontal directions.

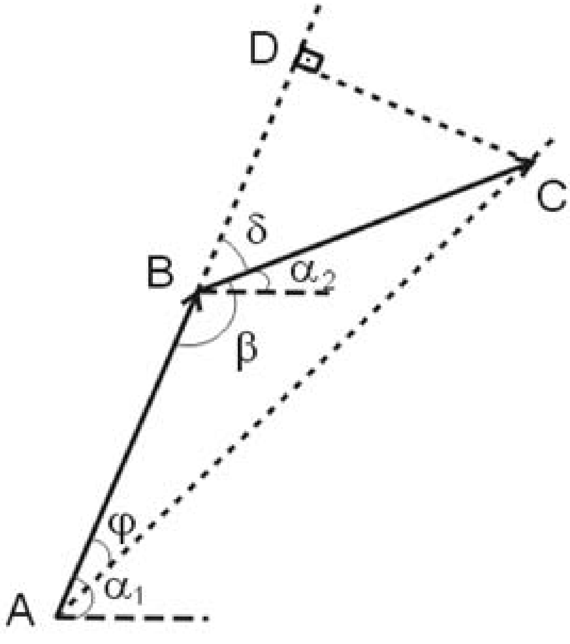
$$sin\,\theta = \frac{G''}{\sqrt{G'^2 + G''^2}}$$

$$cos\,\theta = \frac{G'}{\sqrt{G'^2 + G''^2}}$$

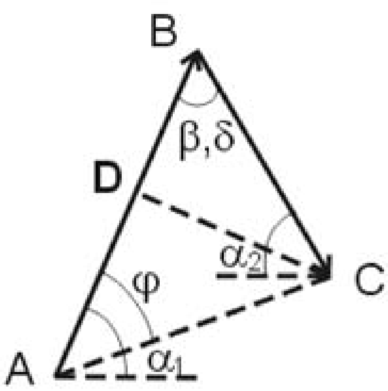


(Jan. 2001-Dec. 2009)

WORK-LIKE AND HEAT-LIKE TERMS



(a)



(b)

$$E_d = \tau\gamma = G\gamma^2 = \frac{\tau^2}{G}$$

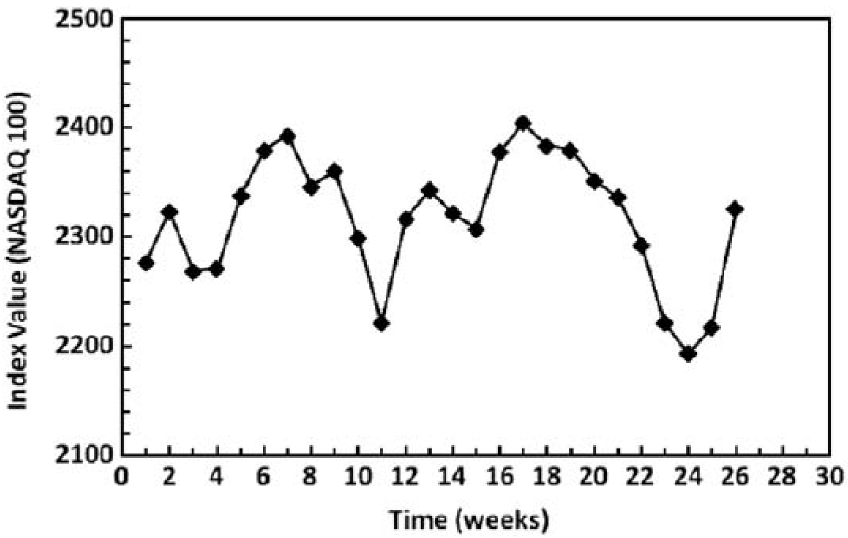
$$E_d = energy\ of\ \ deformation\ (change)$$

$$AB = G' \cdot \tau_{II} \quad \text{and} \quad AB = G'' \cdot \tau_{\perp}$$

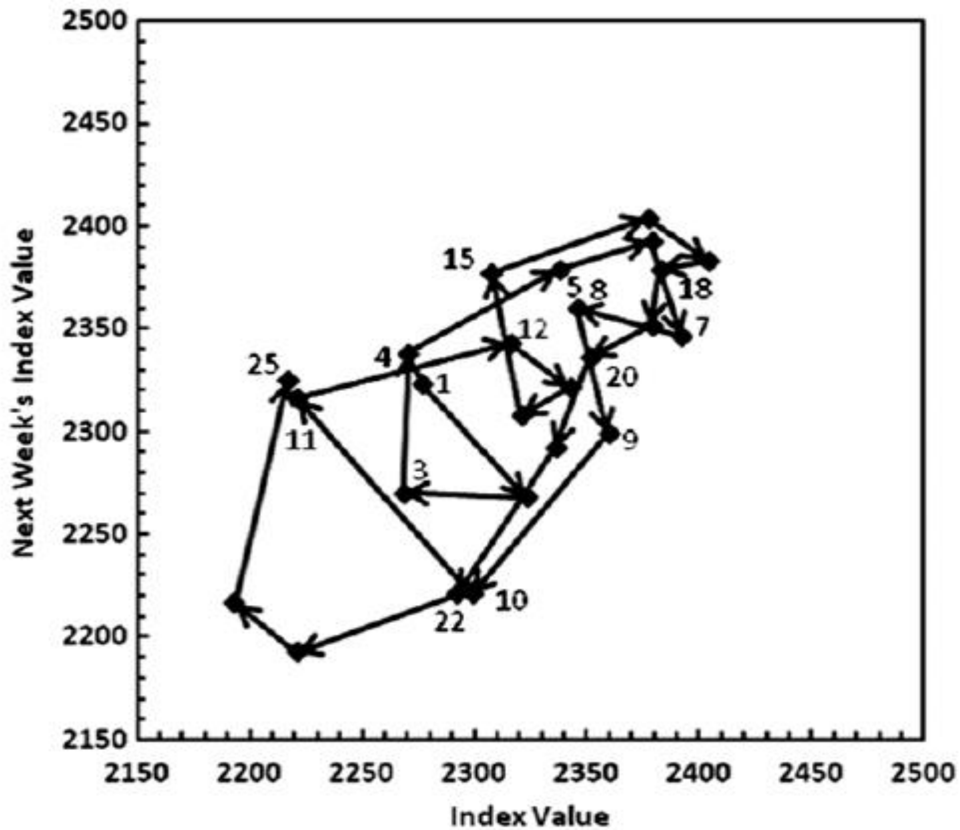
$$W = \overline{AB} \cdot \tau_{II} = \overline{AB} \cdot \frac{\overline{AB}}{G'} = \frac{\overline{AB}^2}{G'}$$

$$q = \overline{AB} \cdot \tau_{II} = \overline{AB} \cdot \frac{\overline{AB}}{G''} = \frac{\overline{AB}^2}{G''}$$

NASDAQ-100



Weekly changes of NASDAQ-100 index (January–June, 2011).



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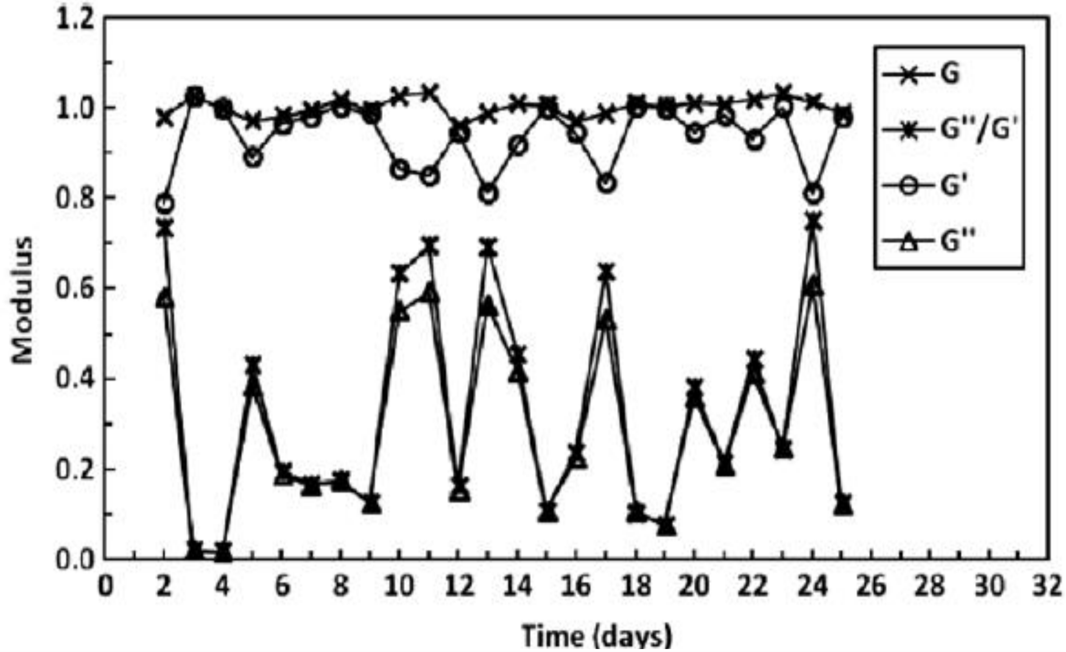
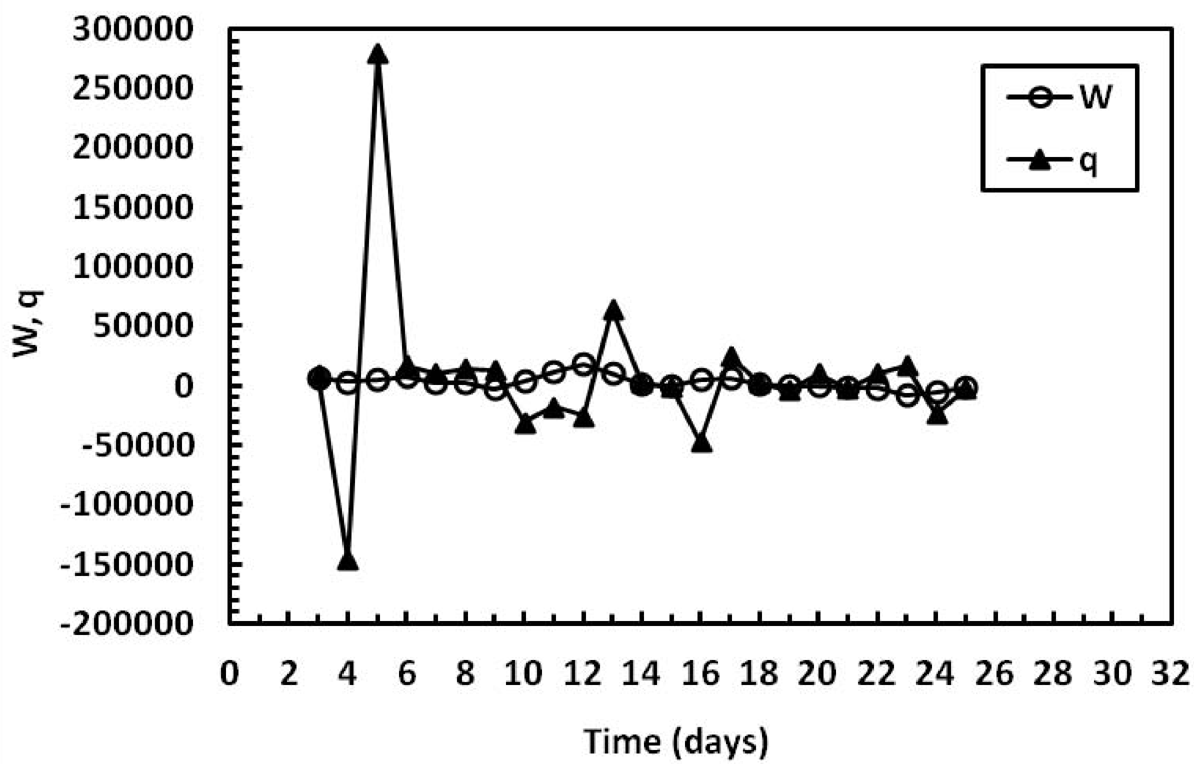
NASDAQ-100





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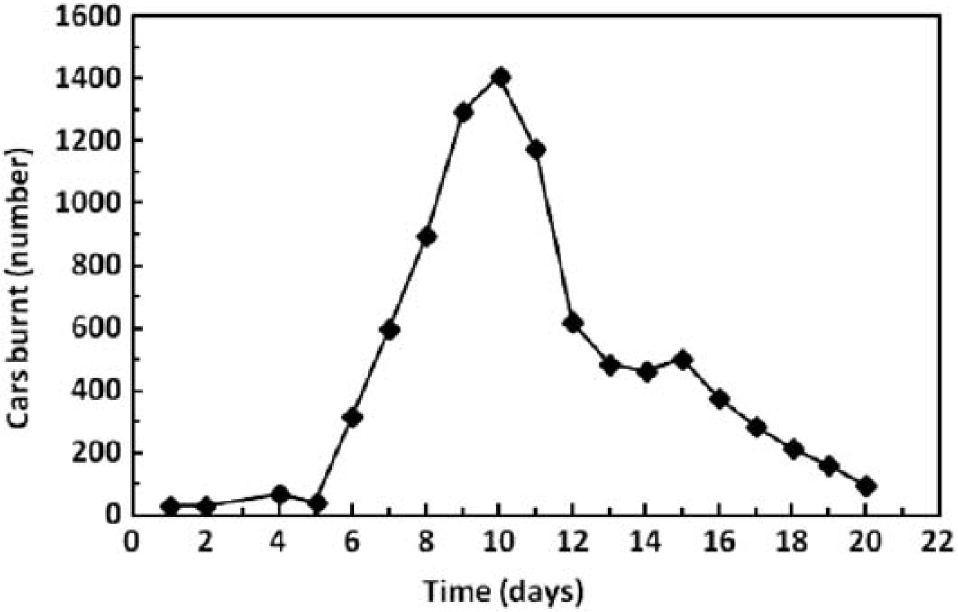
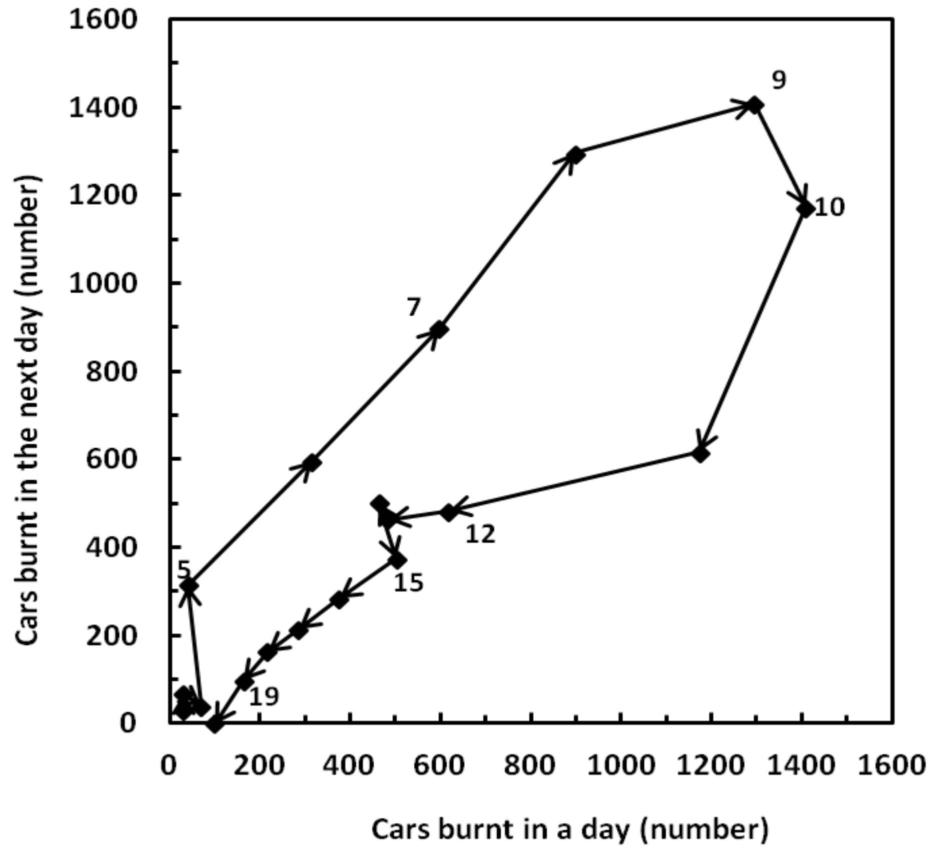
Revolt-Car Burning



Number of cars burnt each day.



(Car Burning Revolt in France)

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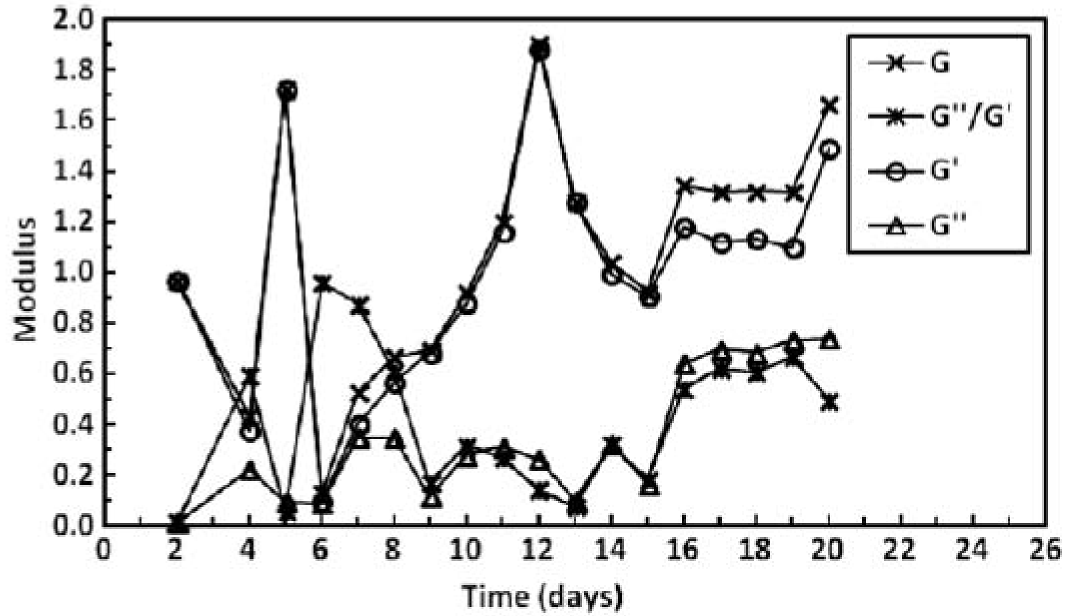
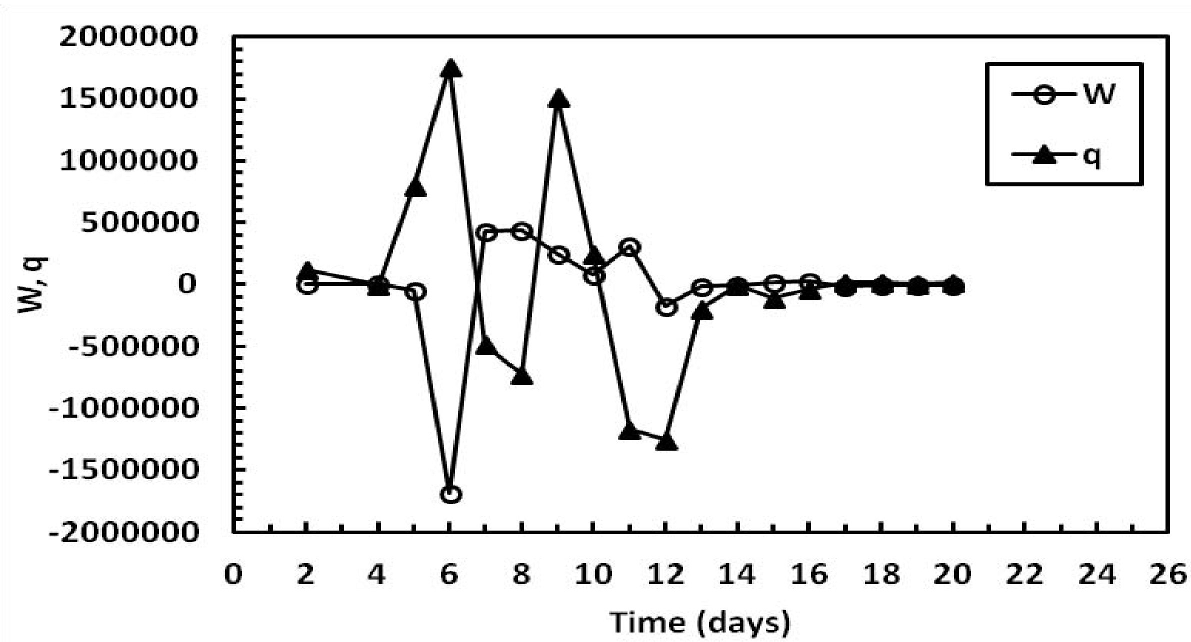
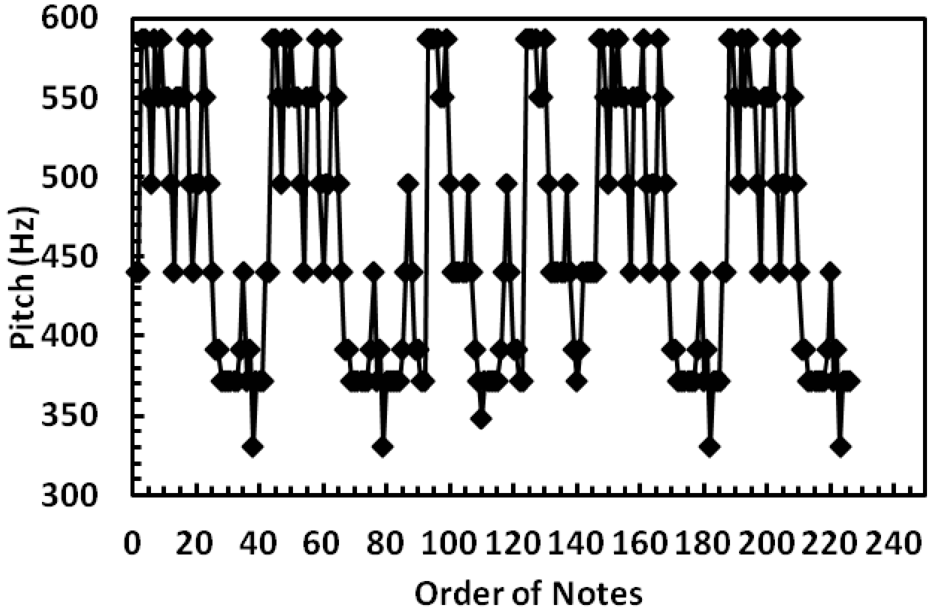
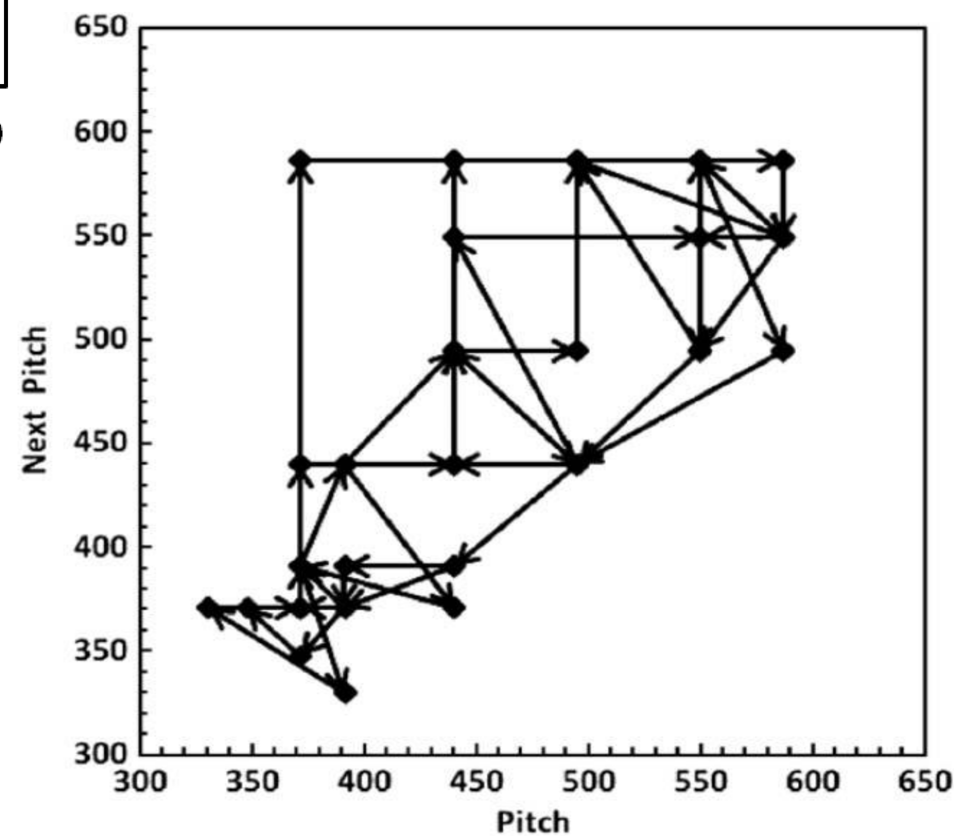
Revolt-Car Burning





Folk Song





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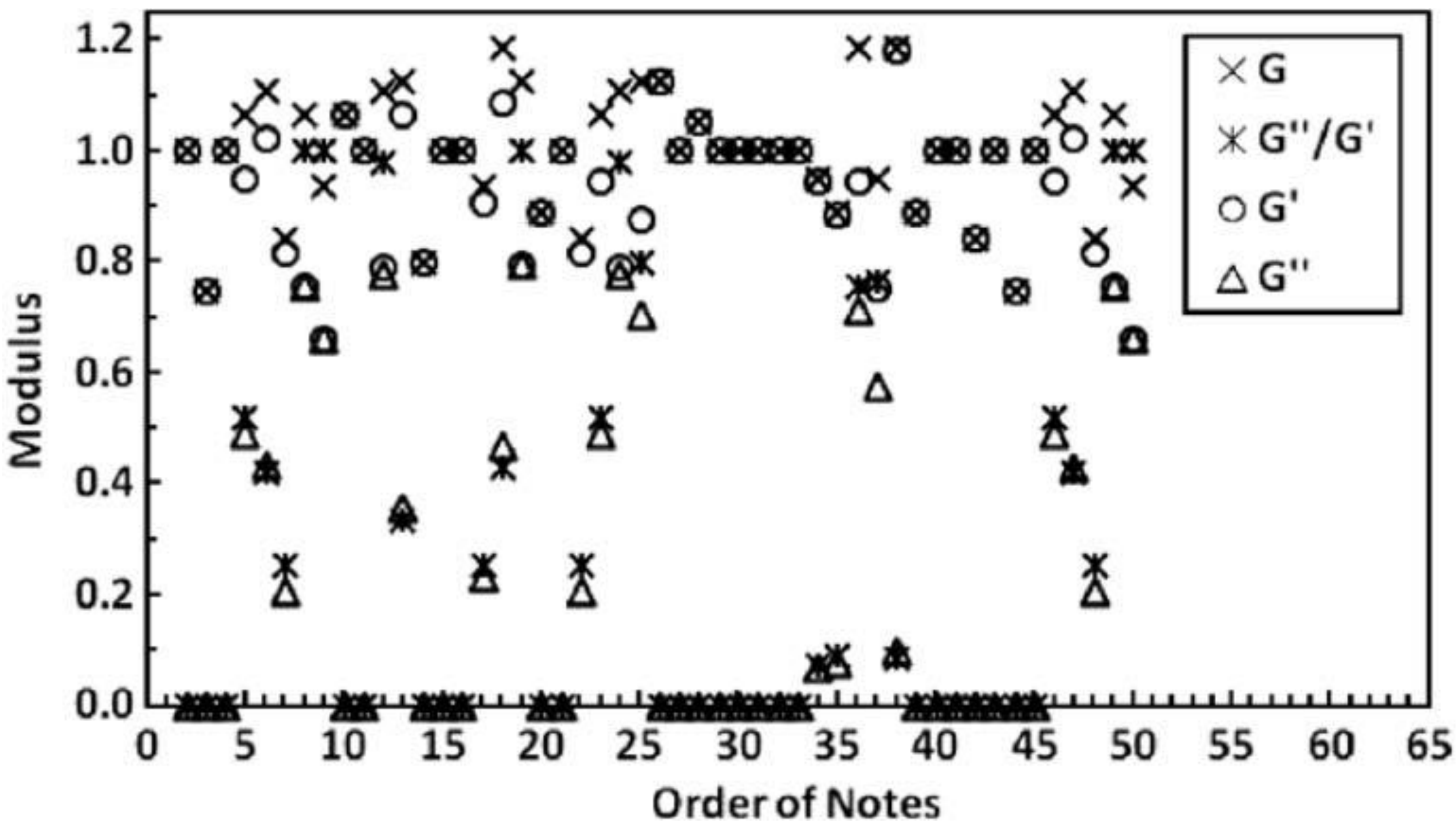
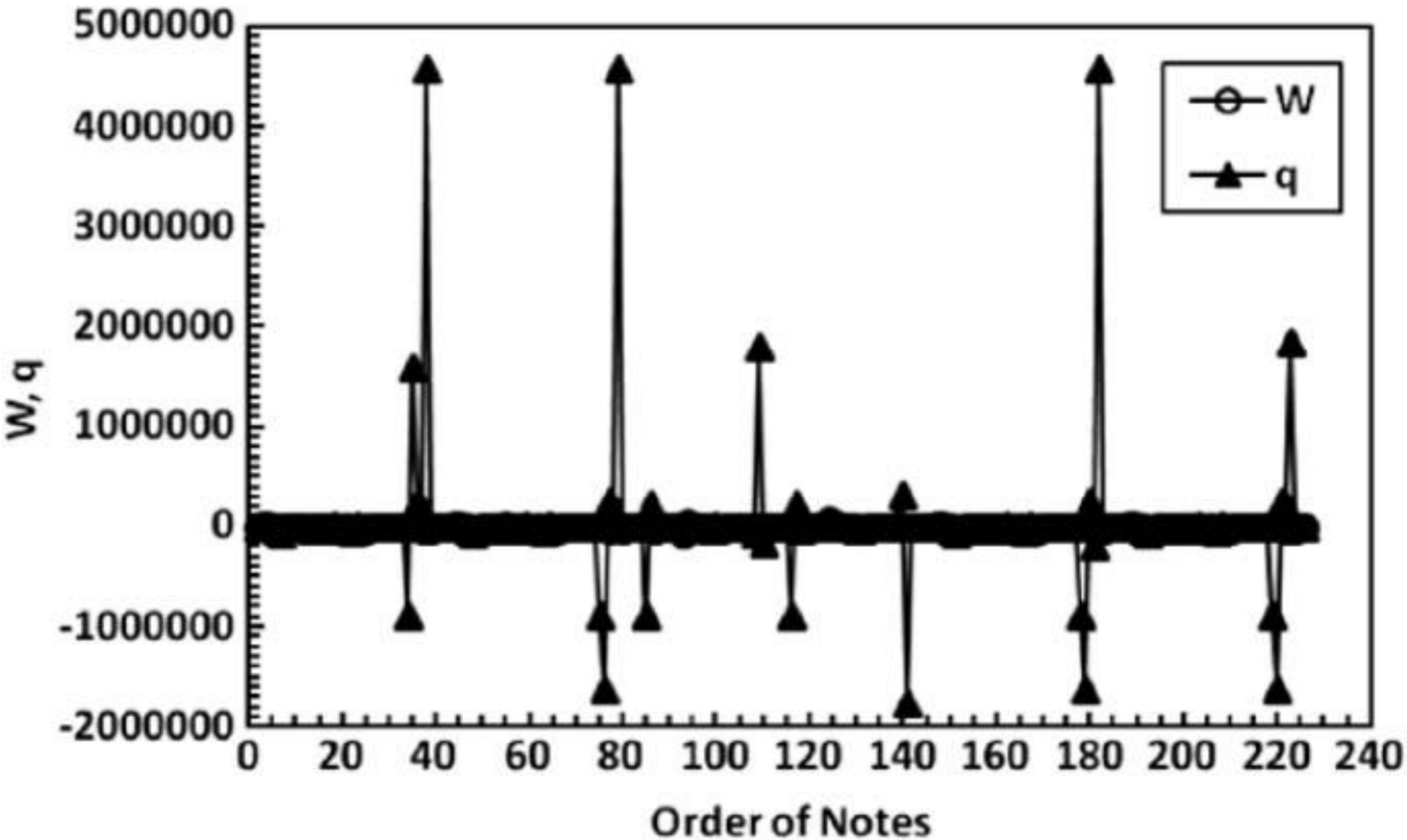
Folk Song



Folk Song



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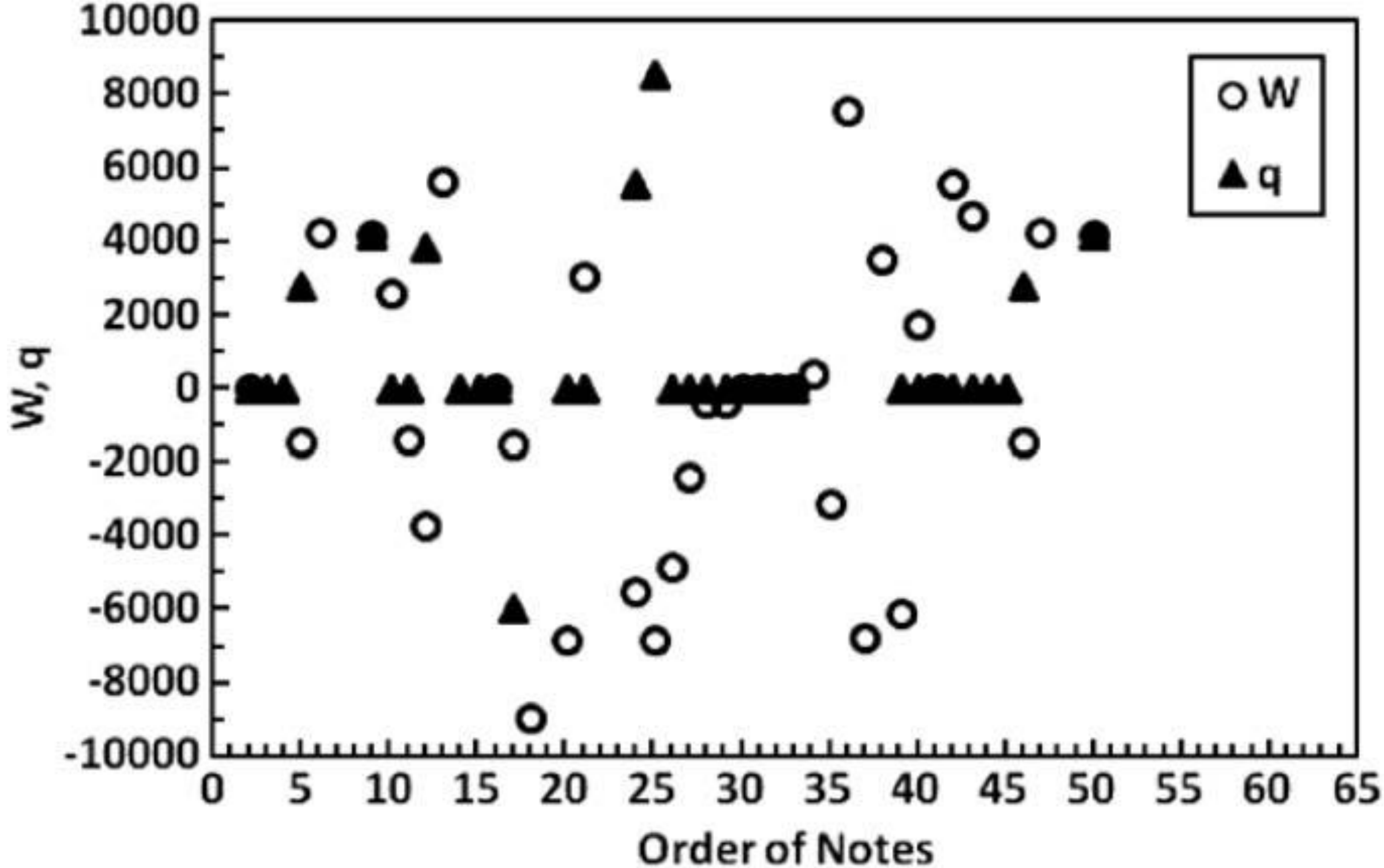
Folk Song



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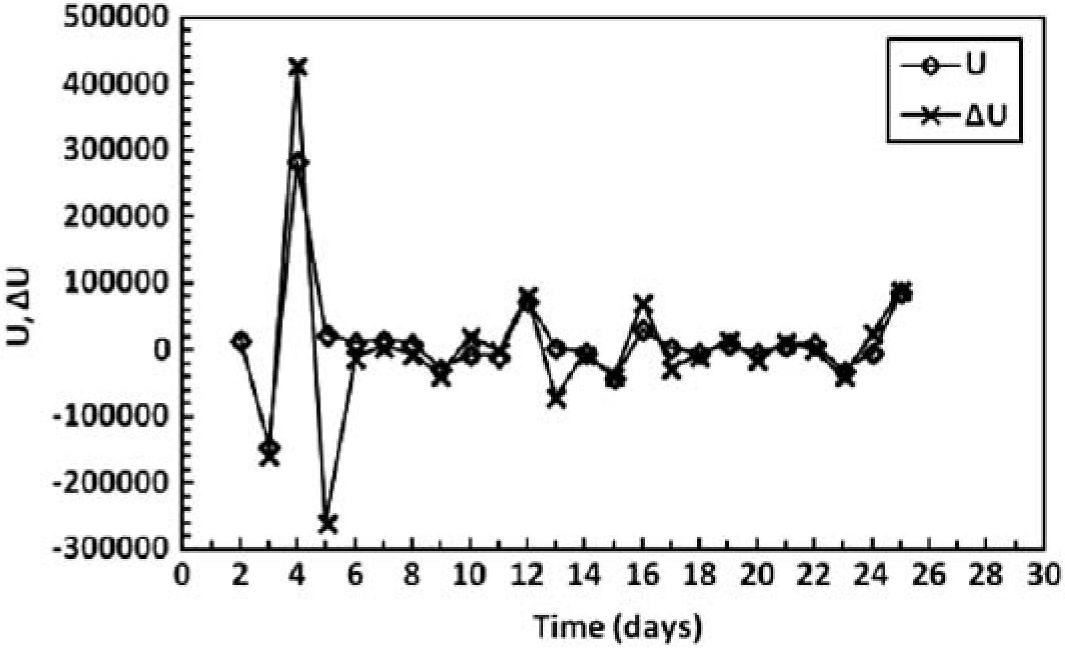
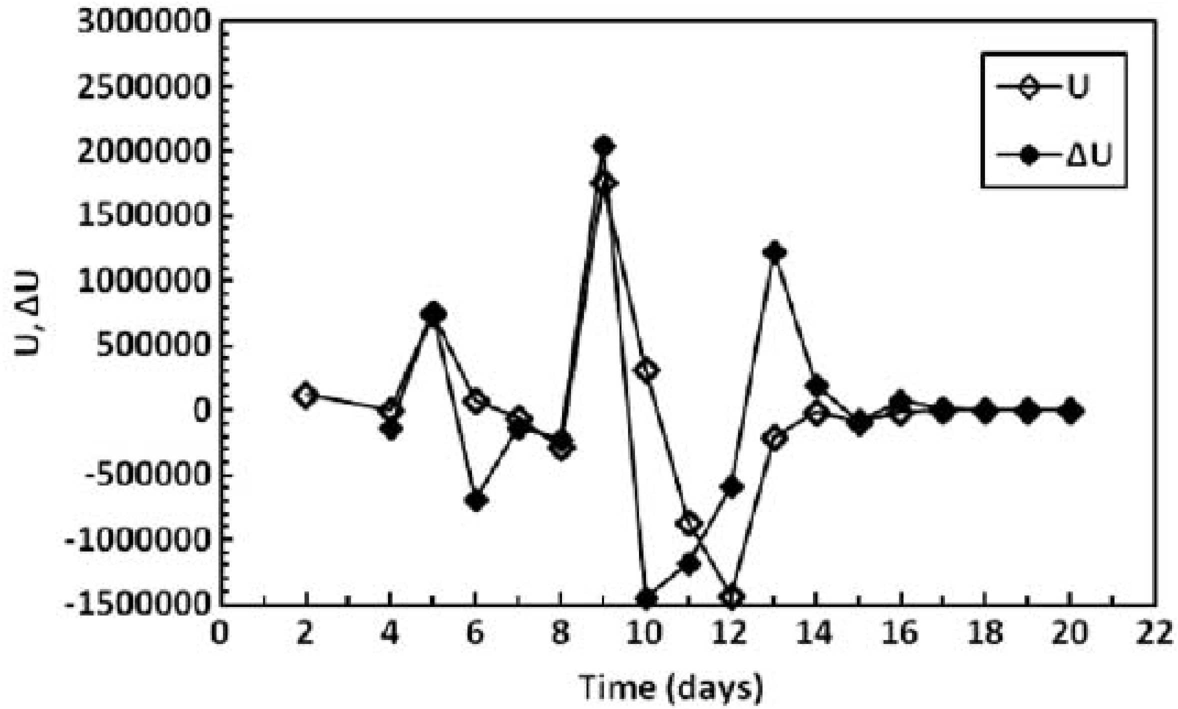


NASDAQ-100.

Car burning



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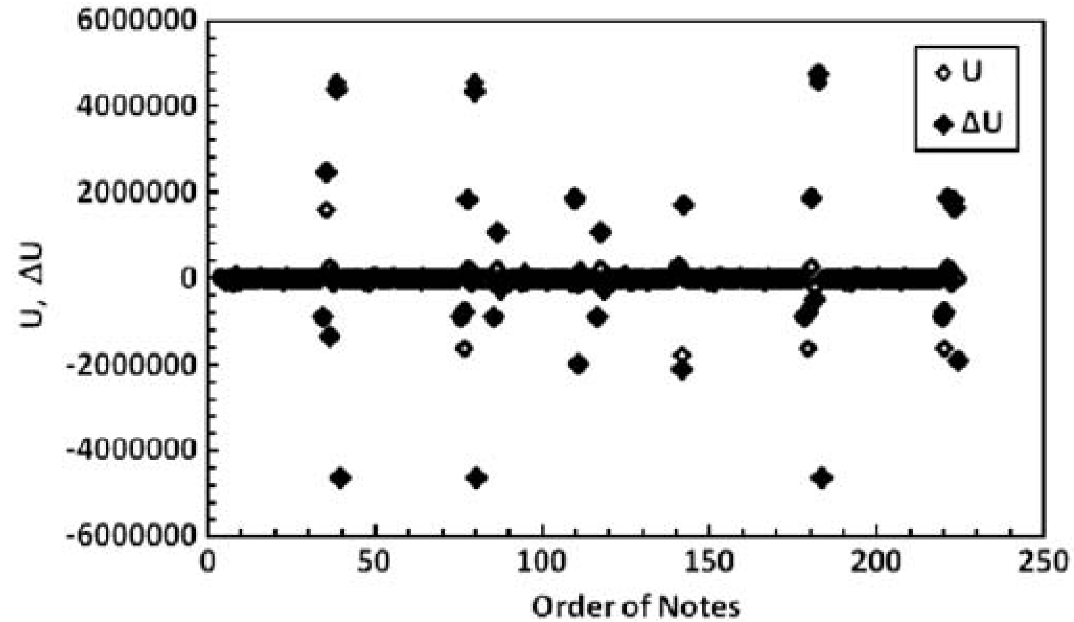
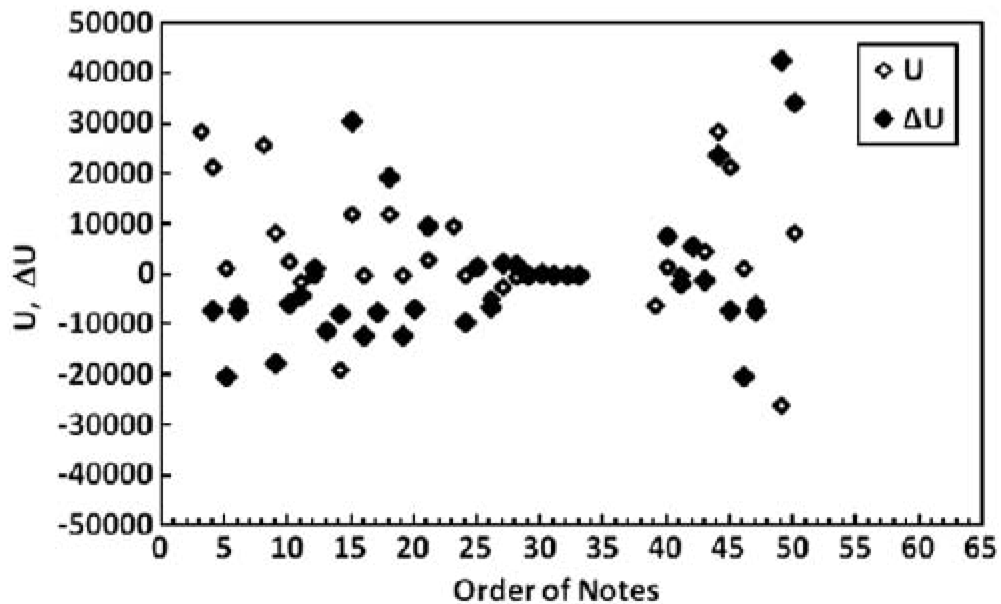
Folk song

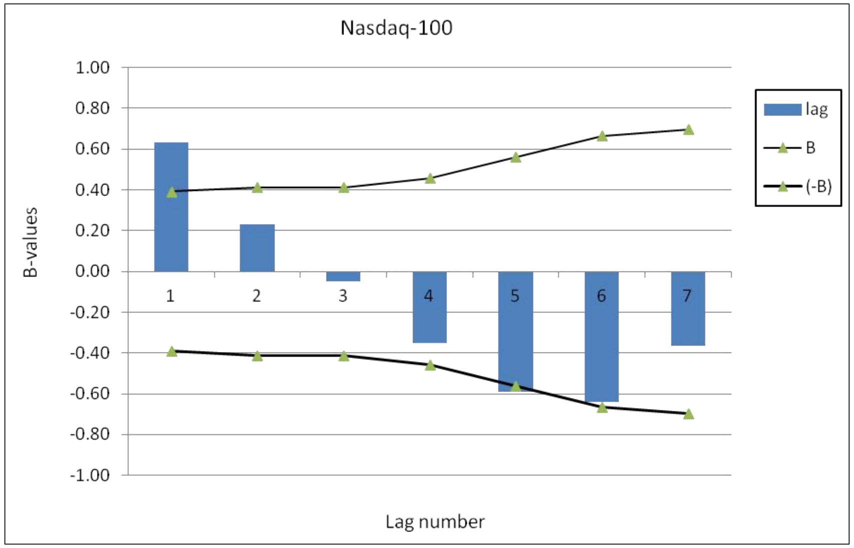


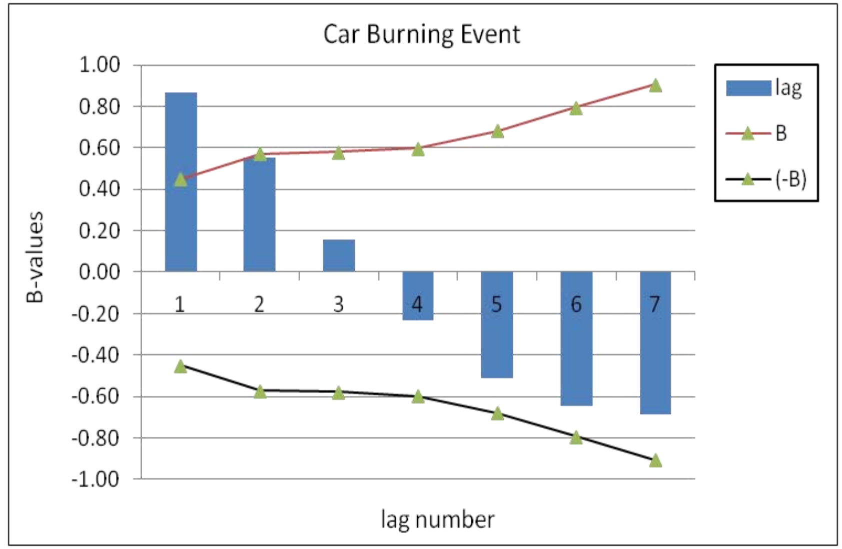


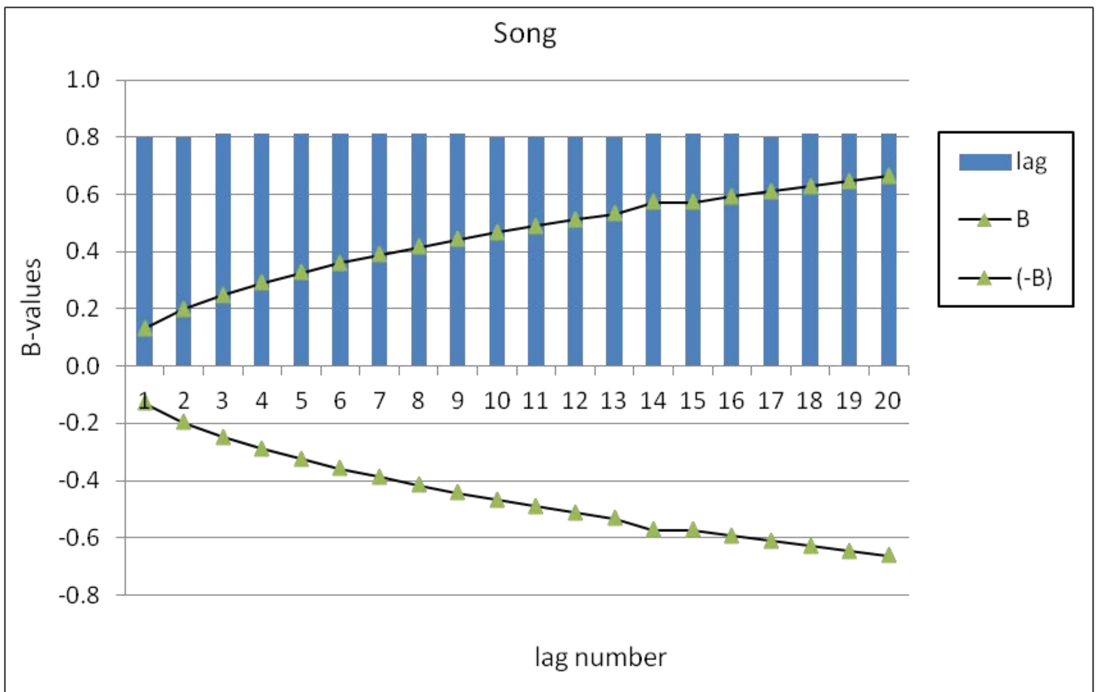
Ulug Bey

Thanks..

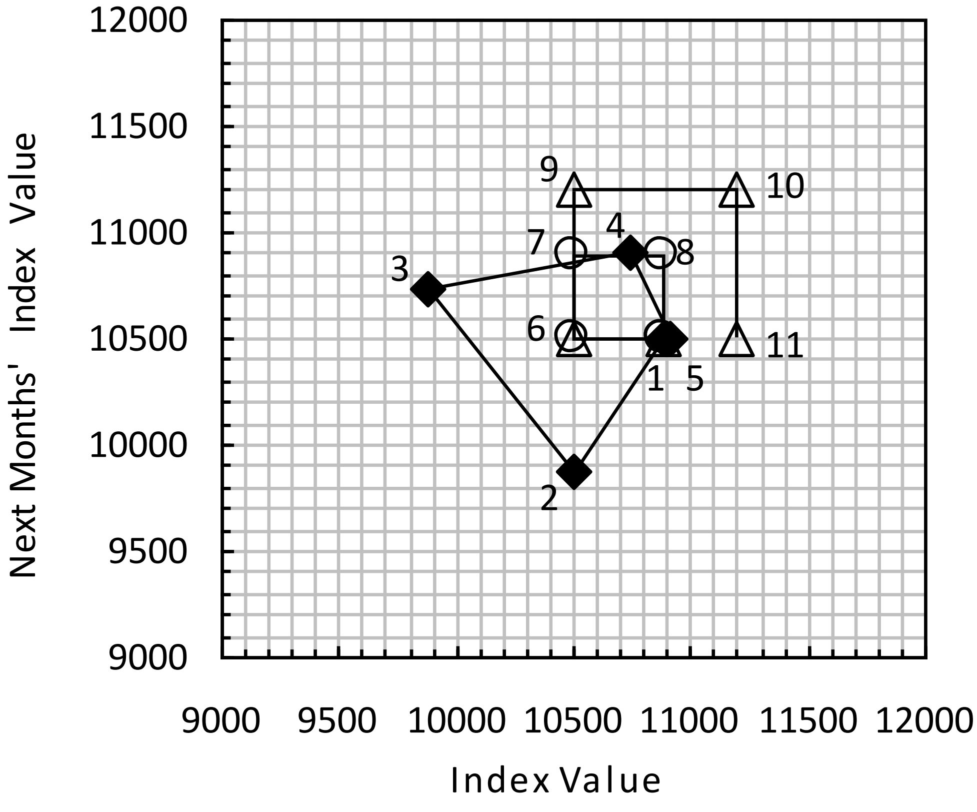
Autocorrelation







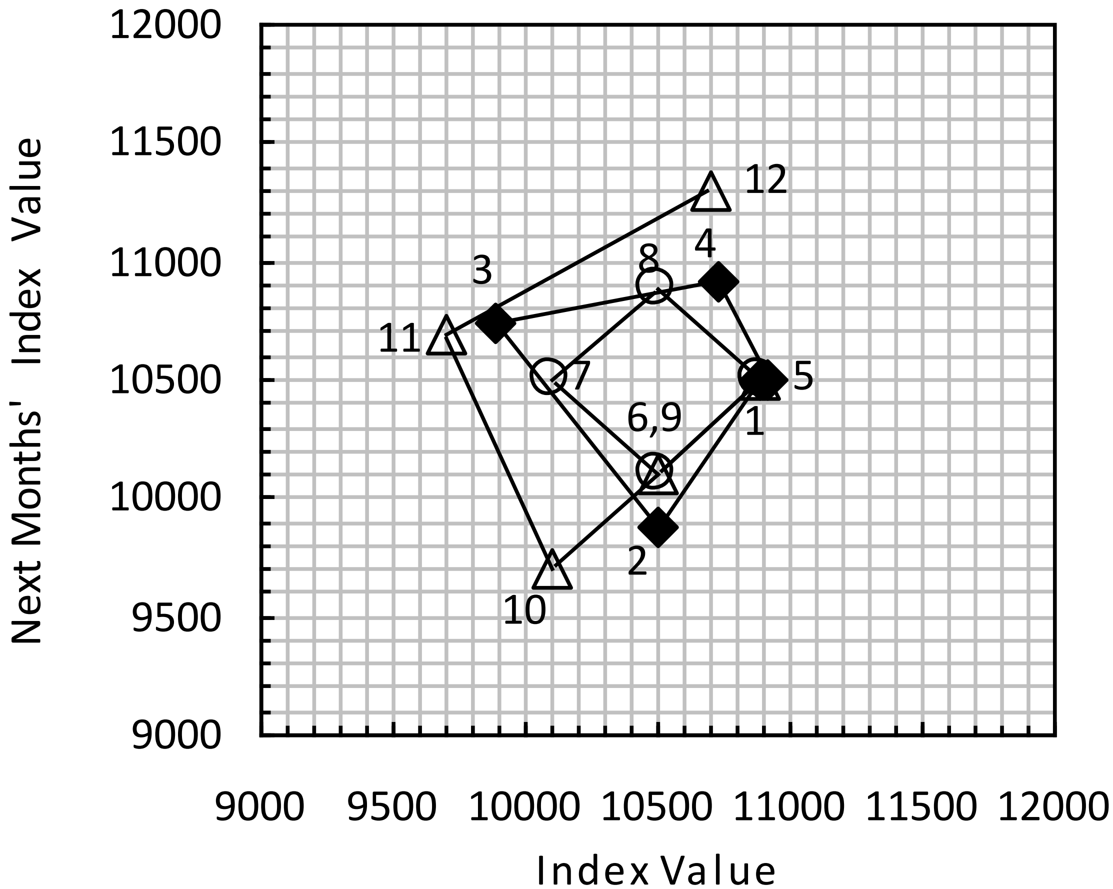
VISCOELASTICITY IN STOCK INDICES



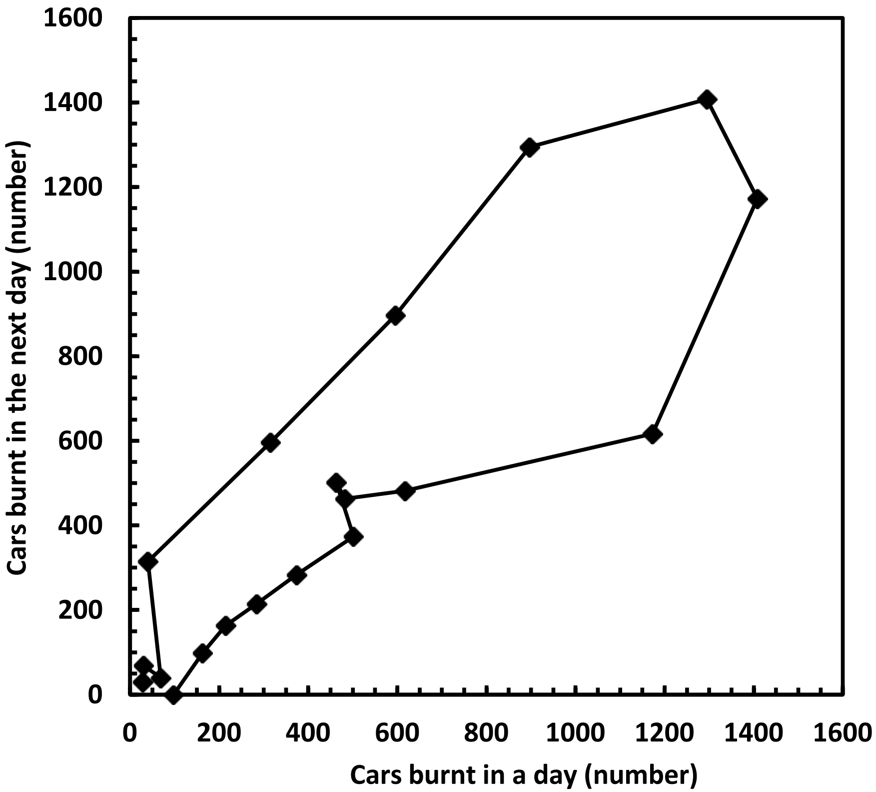
(Scattering Diagram)

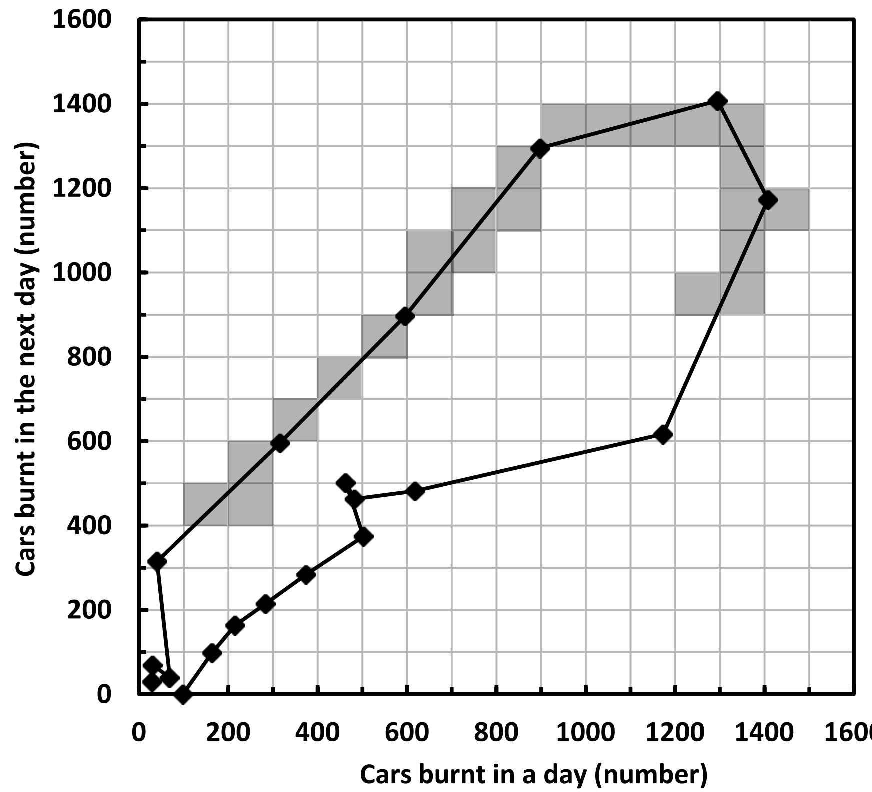
(Data for 1,2,3,4, and 5: DJI, 2001 January – May)

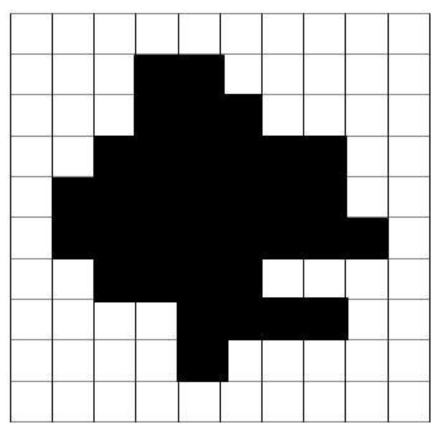
(G. Gündüz and Y. Gündüz, http://dx.doi.org/10.1016/j.physa.2010.09.010)



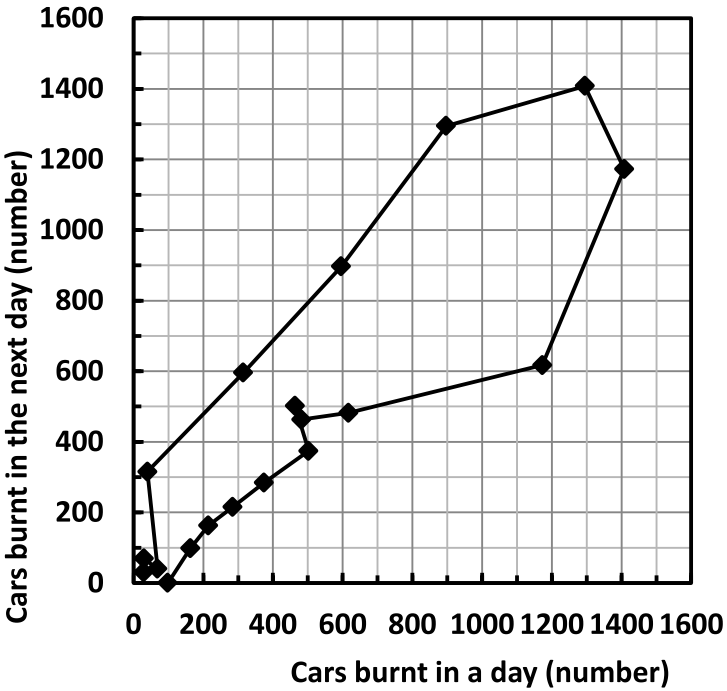
(G. Gündüz and Y. Gündüz, http://dx.doi.org/10.1016/j.physa.2010.09.010)







Animal diagram
(Lattice animal)



(Car Burning Revolt in France)