Universality in Seismicity Statistics and Interim Report on the 3.11 EQ. (2011) in Fukushima-Miyagi Area

Yoji Aizawa

Part 0. Outline of the 3.11 EQ.

Part Ⅰ. Empirical Laws in Seismicity statistics

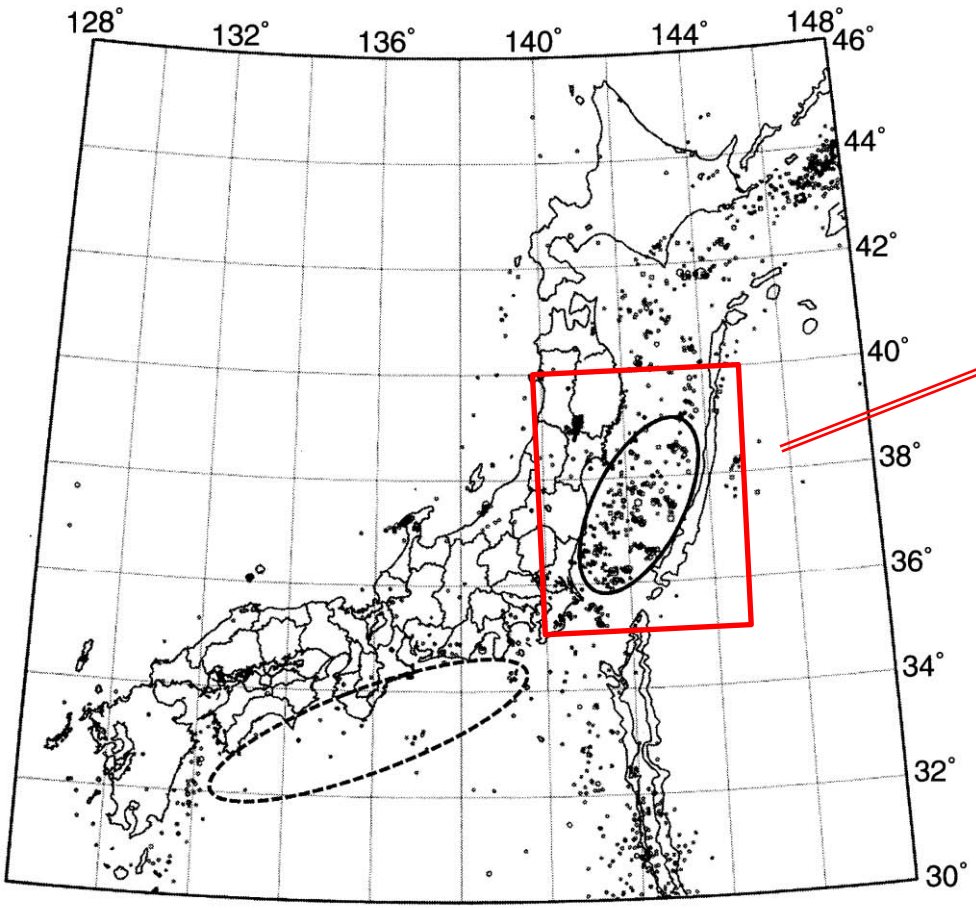
- New aspects obtained from the 3.11 EQ. -

Part Ⅱ. Multi-fractal relation and unified formulae

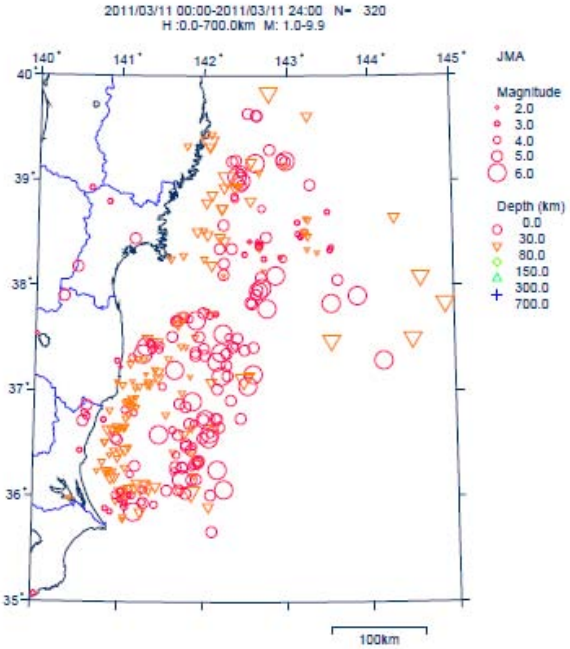
Part Ⅲ. Conclusions and prospects

EQs : 2006-2010

(Hypocenter, depth < 60km, M>4.0)



Aftershocks in Fukushima-Miyagi area
(3.11 – 11.11, 2011)



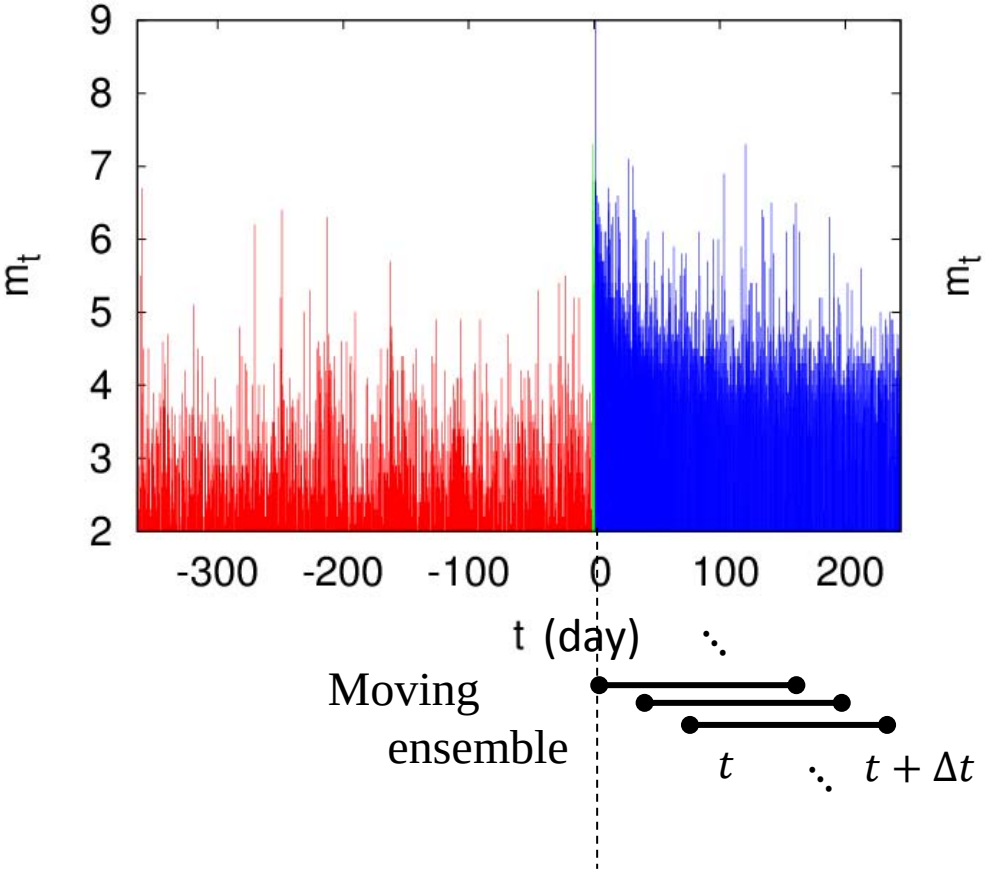
(JMA)

(2011, T. Matsuzawa)

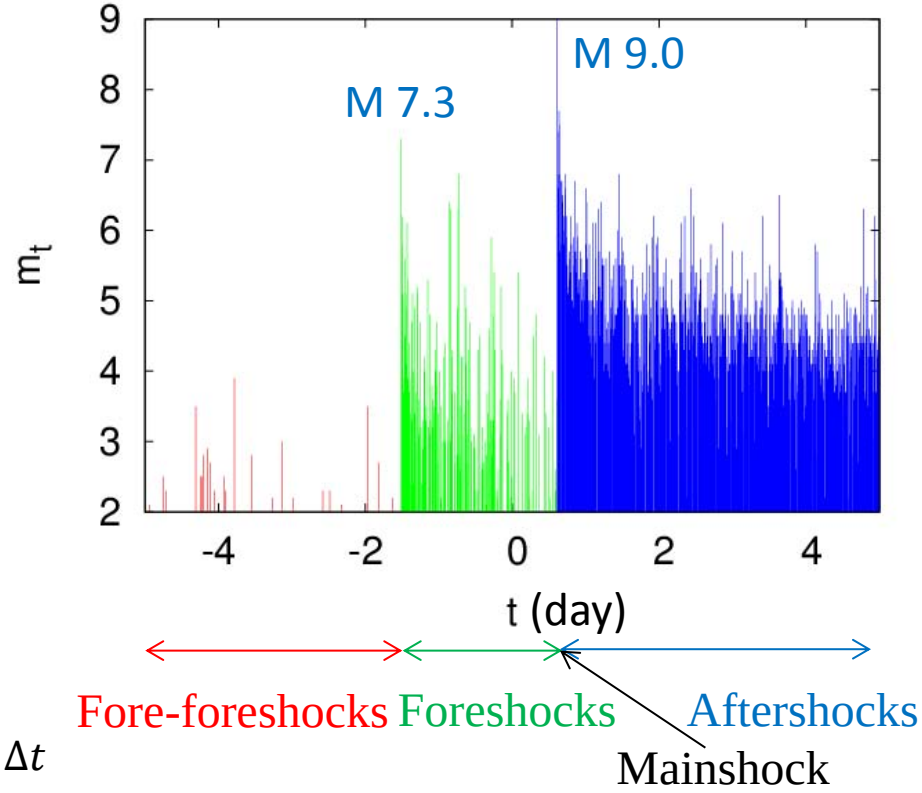
Sequence of EQs (in Magnitude)

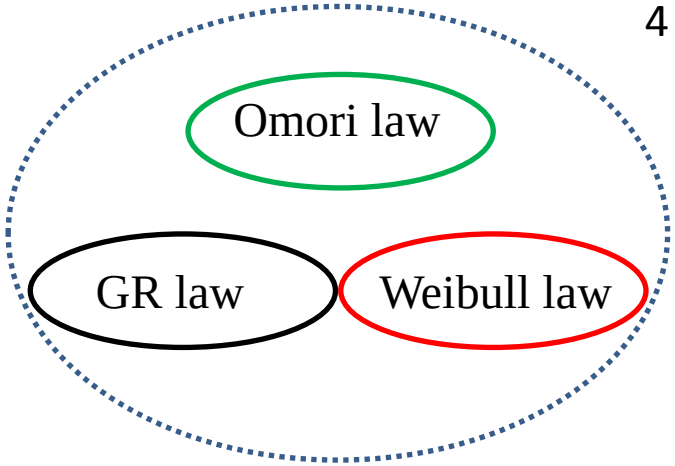
Before and After 3.11

2010/3/11～2011/11/11



2011/3/06～2011/3/16





Part I. Empirical Laws in Seismicity Statistics

- New aspects obtained from the 3.11 EQs -

1. Omori formula in Aftershocks' Sequence
2. Magnitude, Energy, Amplitude Distributions (Gutenberg-Richter, Wadati, Ishimoto-Iida)
3. Interoccurrence time distributions (Weibull law)

Empirical laws in seismicity statistics

1. Omori formula (1894)

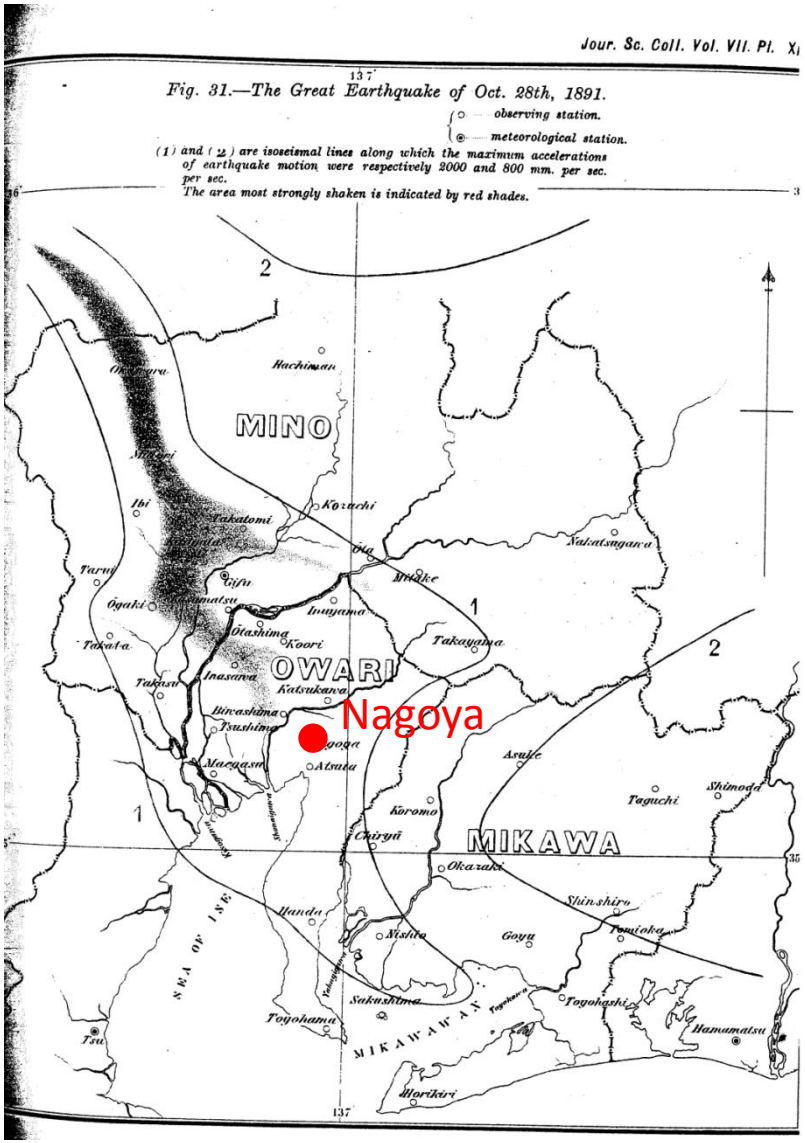
$N(T)$: cumulative numbers of aftershocks (for $0 \leq t \leq T$)

$$\boxed{N(T) \propto \log(aT + b)}$$

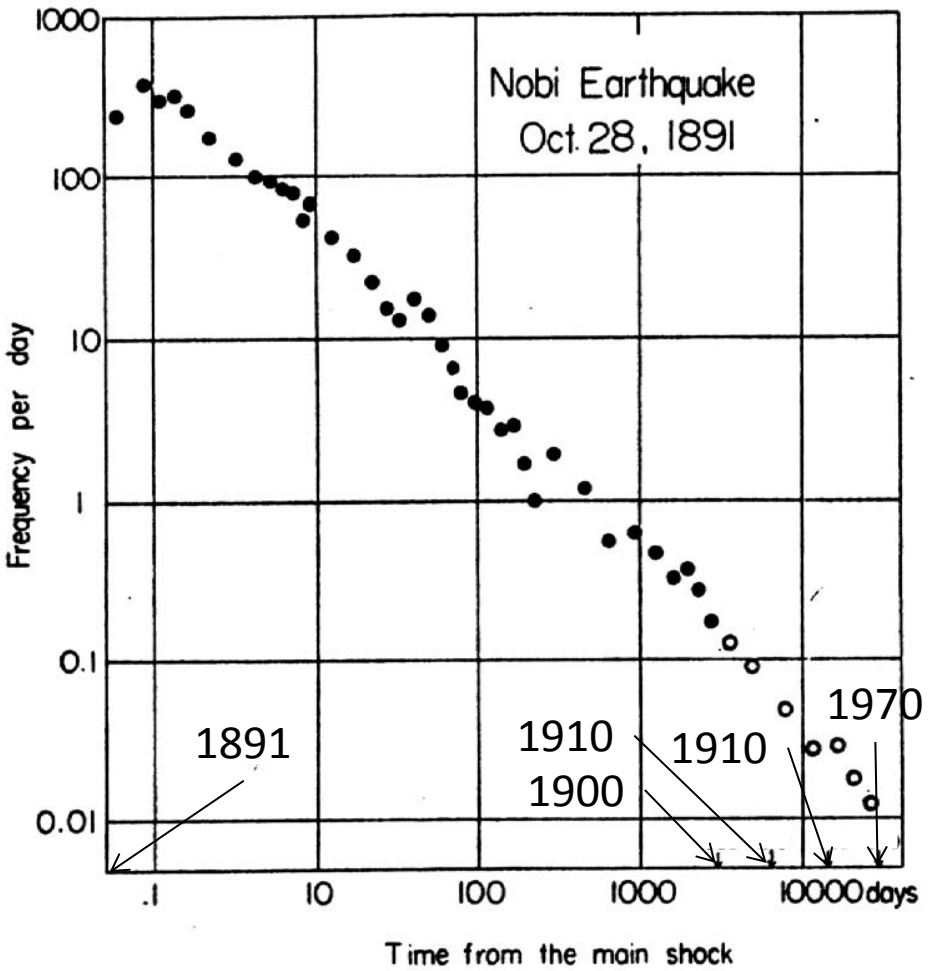
$n(T)$: aftershock frequency per unit time (1 day)

$$\boxed{n(T) = \frac{dN(T)}{dT} \propto \frac{1}{(aT + b)^c} \, (c \cong 1)}$$

Empirical laws in seismicity statistics (Nobi EQ, m.8.4, 1891)



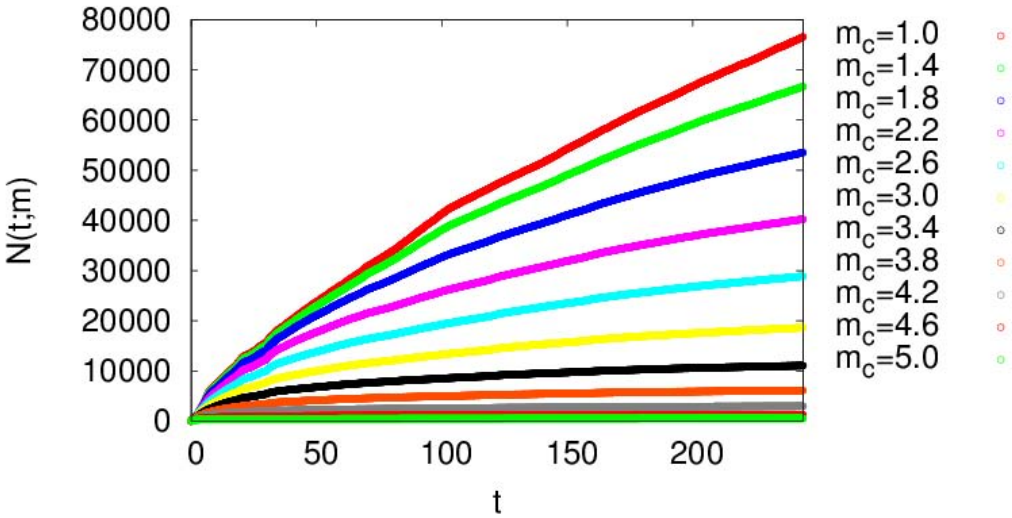
Omori formula (Utsu, 1969)

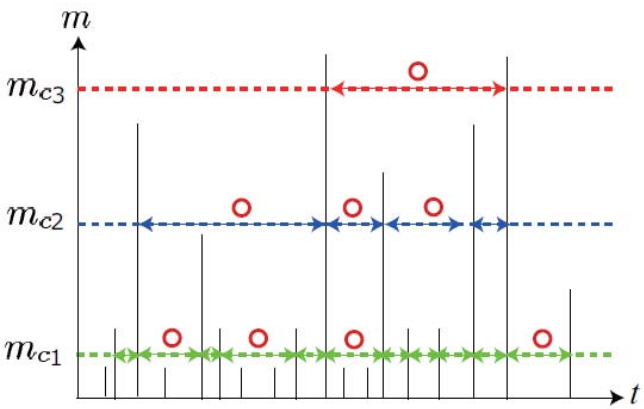


Empirical laws in seismicity statistics (3.11 EQ)

Generalization of Omori formula (Aftershocks of 3.11 EQ)

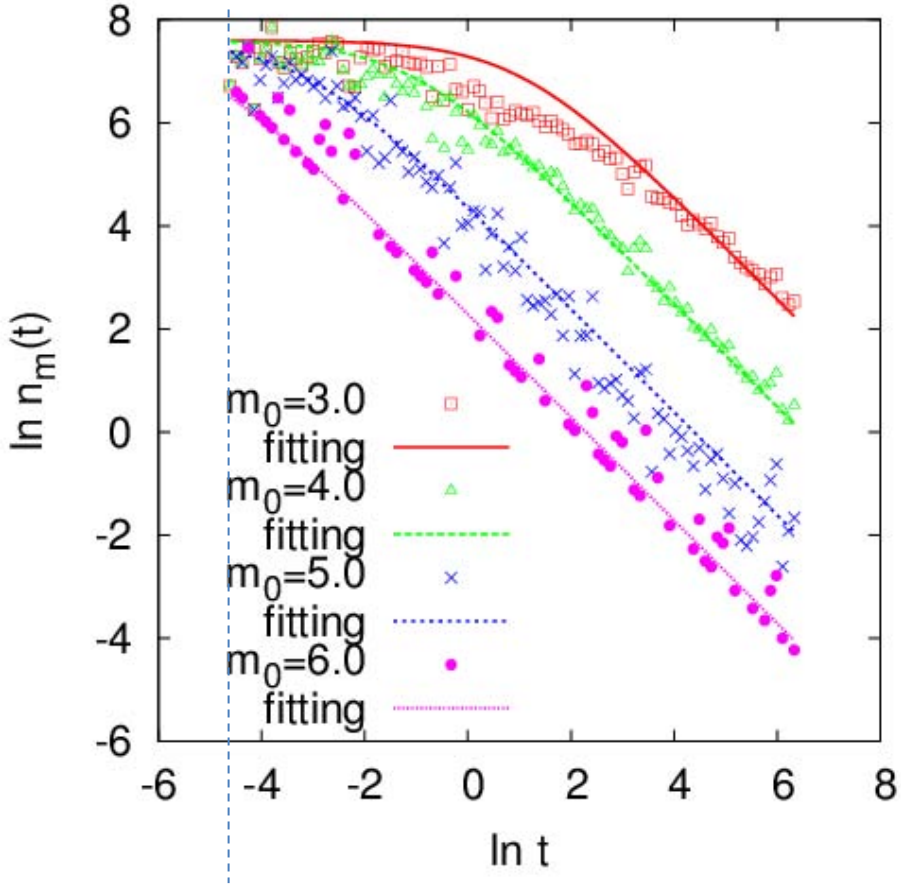
$$N(t;m) = dC_m \log(c_m^{-1}t + 1)$$





Empirical laws in seismicity statistics (3.11 EQ)

2011/3.11 – 2012/11.11
(#EQ=73576, $M \geq m_0 = 2.0$)



10(sec)

$$n_m(t) = \frac{d}{1 + t c_m^{-1}},$$

$$\ln d \cong 7.6, c_m \cong e^{-\hat{b}(m-3.5)}$$
$$\hat{b} = 2.10$$

Extrapolation formula of the GR law:
$P_m(t)$:Cumulative magnitude distribution

$$P_m(t) = \frac{n_m(t)}{n_{m_0}(t)} = e^{-b(m-m_0)} \frac{t + c_{m_0}}{t + c_m}$$
$$(m_0 (m \gtrsim 3.5): min. cutoff\ mag.)$$

at $t \to \infty$ (stationary regime)

$$P_m = e^{-b(m-m_0)} \quad (GR\ law)$$

Empirical laws in seismicity statistics (GR, Wadati, Ishimoto-Iida)

2. Intensity of EQs

i) Magnitude (m) distribution of EQs

$$\log_{10} E \fallingdotseq 1.5m + 11.8$$

Gutenberg-Richter formula (1943, 1956)

$$\boxed{P(m) \propto 10^{-b_{10}m}\left(= e^{-bm}\right)} \quad \left(\mathrm{b} \cong \frac{1}{< m - m_0 >}\right)$$

ii) Energy (E) distribution of EQs

Wadati formula (1932)

$$\boxed{P(E) \propto E^{-k}}$$

iii) Maximum amplitude (a) distribution of EQs

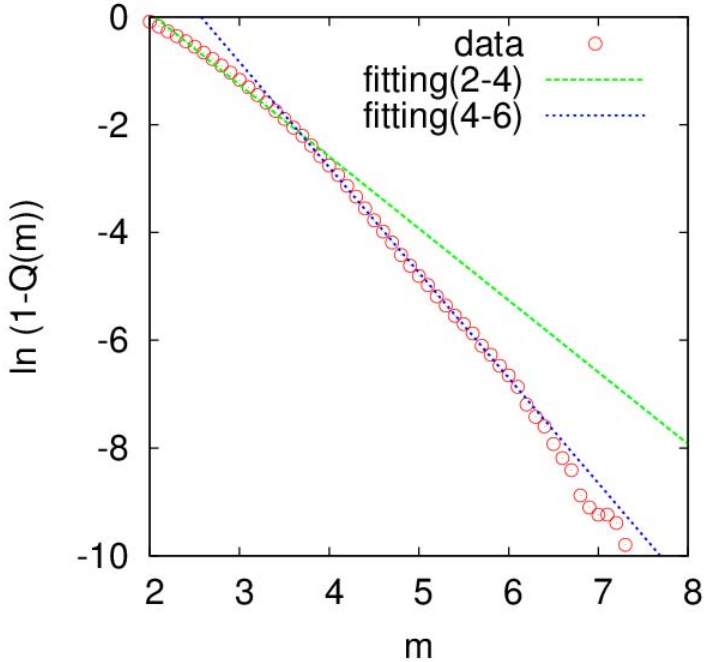
Ishimoto-Iida formula (1939)

$$\boxed{P(a) \propto a^{-k\prime}}$$

Empirical laws in seismicity statistics (GR law in 3.11 EQ)

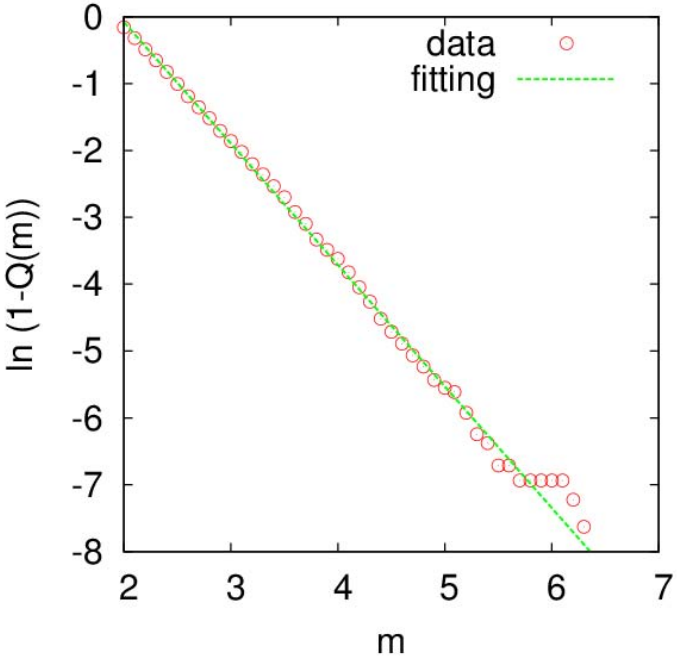
GR formula (3.11 EQ): $P(m) = b\exp[-bm]\,, Q(m) = 1 - e^{-bm}$

Aftershocks



Fitting[2:4] $b = 1.3$
Fitting[4:6] $b = 1.9$

Fore-foreshocks



$b = 1.8$

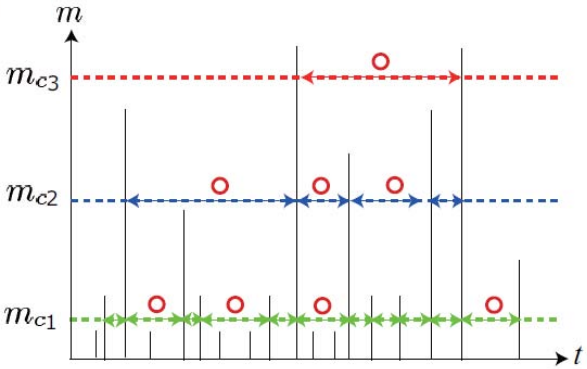
$b \cong 1.8\sim2.0$ (by Generalized Omori law)

Empirical laws in seismicity statistics

3. Interoccurence time (τ) distribution : $P(\tau, m)$

Candidates studied so far ;

$$P(\tau, m) = \begin{cases} \text{Weibull} \\ \text{Gamma} \\ \text{Log} - \text{Weibull} \\ \text{Inverse} - \text{Power} \\ \text{Log} - \text{Normal} \end{cases}$$



Our results (2001-2007, Japan, California, Taiwan)

$P(\tau, m) =$ Weibull D + Log-Weibull D

(Dominant) (Minor Correction)

Goodness -Fit-Test
- RMS Test
- Kormogorov-Smirnov Test
- Anderson-Darling Test

Cumulative Prob.

$$Q(\tau; m) = 1 - e^{-\left(\frac{\tau}{\beta_m}\right)^{\alpha_m}}$$

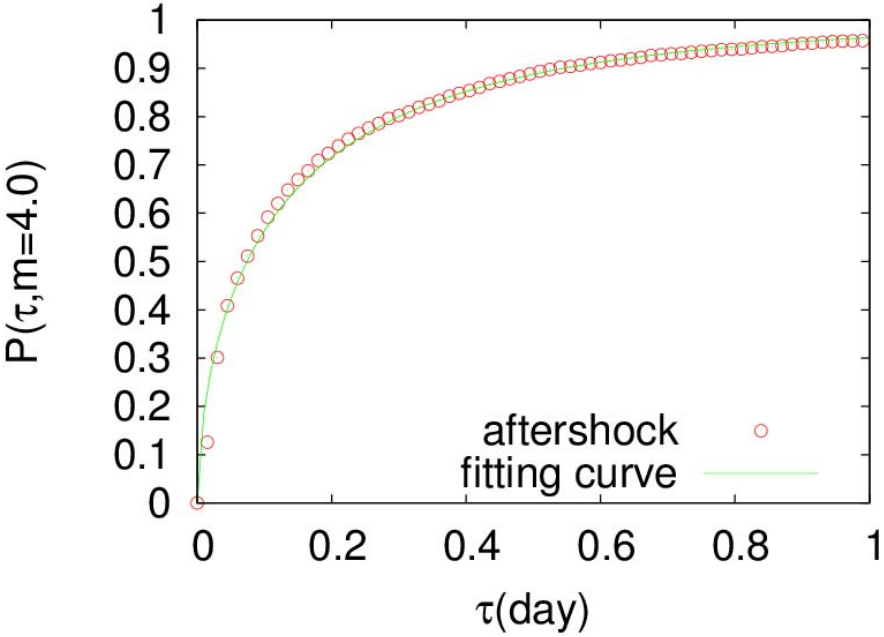
(Ref. Tectonophys, 2010)

Empirical laws in seismicity statistics (3.11 EQ)

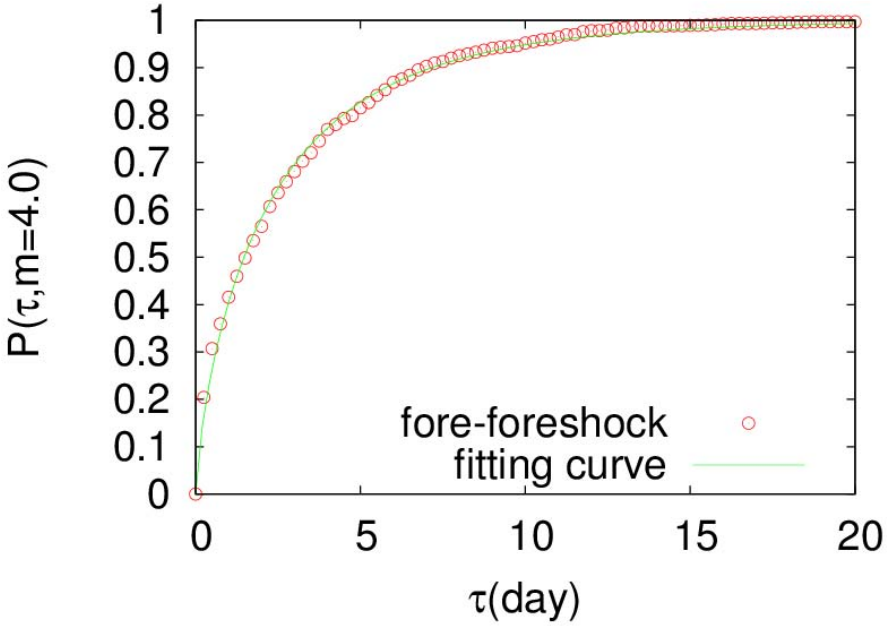
Interoccurence time distribution (3.11 EQ):

$$P(\tau;m) = \frac{\alpha_m}{\beta_m}\left(\frac{\tau}{\beta_m}\right)^{\alpha_m - 1} \exp[-\left(\frac{\tau}{\beta_m}\right)^{\alpha_m}], Q(\tau,m) = 1 - \exp[-\left(\frac{\tau}{\beta_m}\right)^{\alpha_m}]$$

Aftershocks (for 20 months)



Fore-foreshocks (for 10 years)

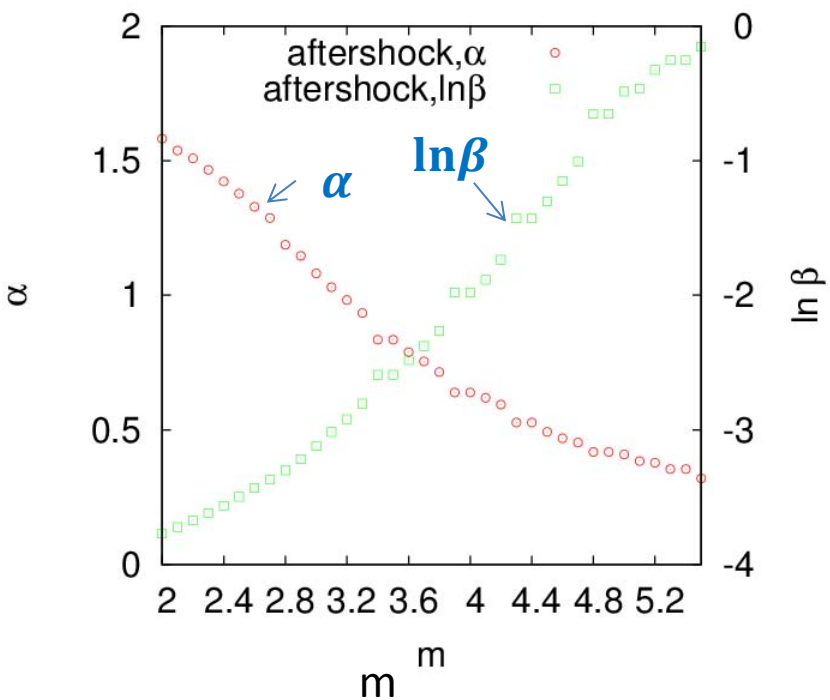


Empirical laws in seismicity statistics (3.11 EQ)

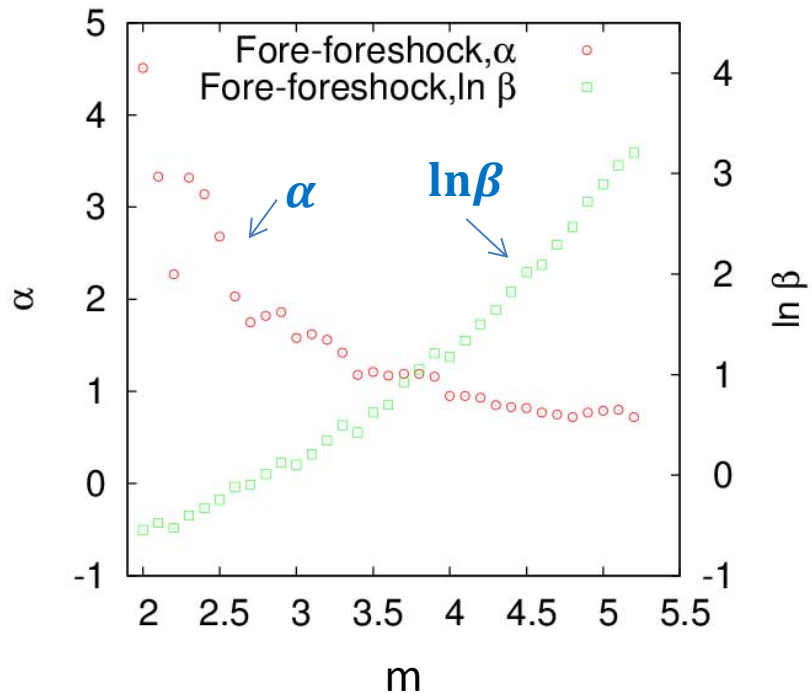
Magnitude-dependence of Weibull parameters α and β (3.11 EQ)

Multi-fractal relations ; $\begin{cases} \alpha = f_\alpha(m,b) \\ \beta = f_\beta(m,b) \end{cases} \longrightarrow b = g(\alpha,\beta)$

Aftershocks (for 20 months)

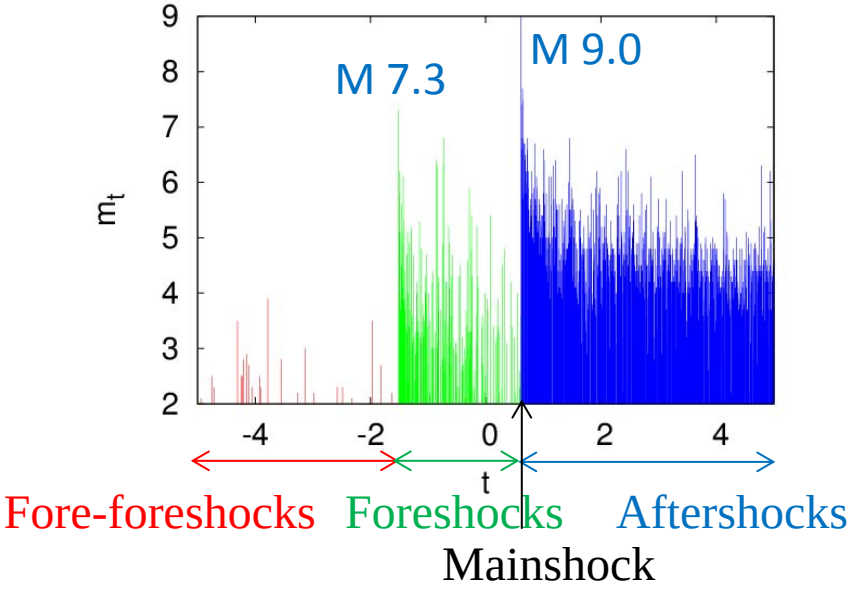


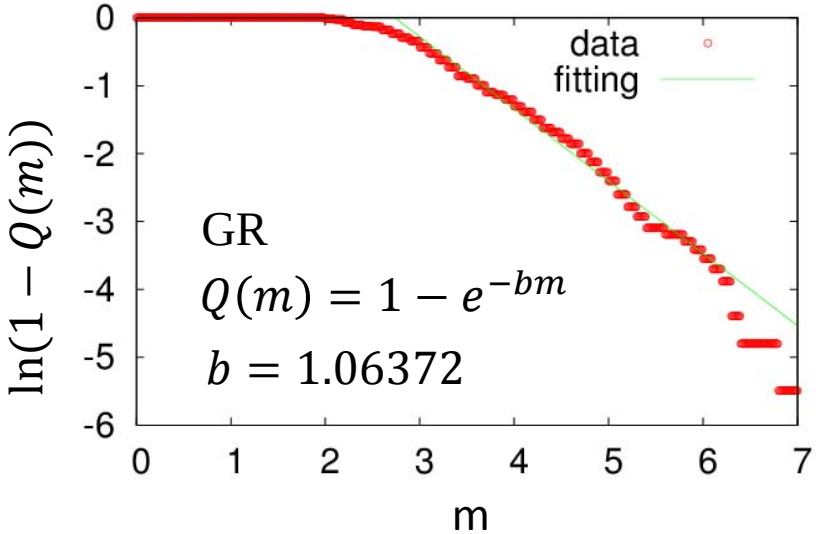
Fore-foreshocks (for 10 years)



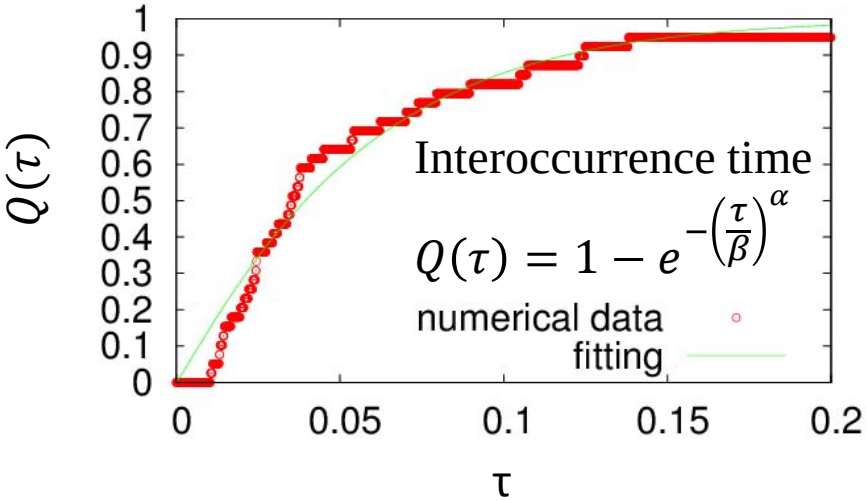
Empirical laws in seismicity statistics (Foreshocks of 3.11 EQ)

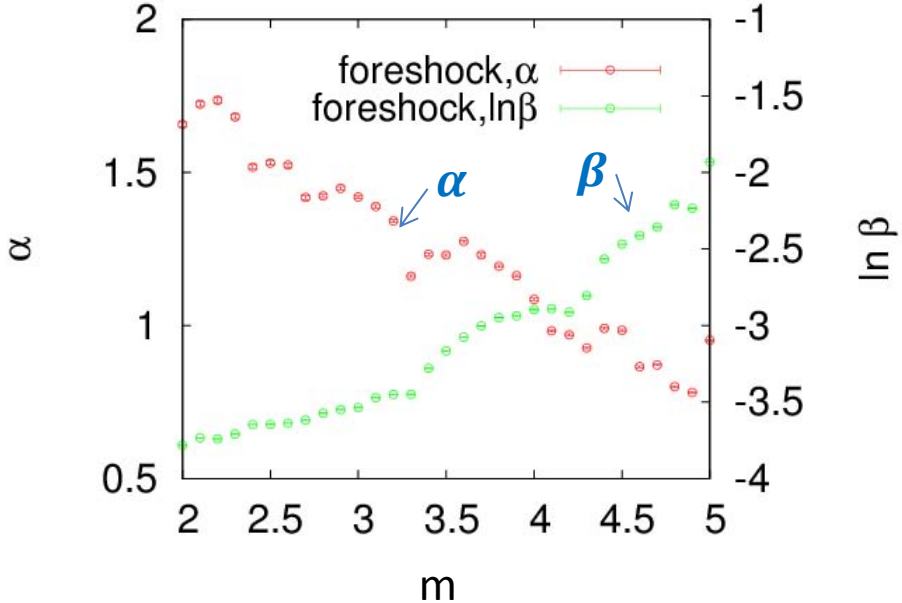
Foreshocks; 3.09-3.11(2011), # EQ = 470





m=4



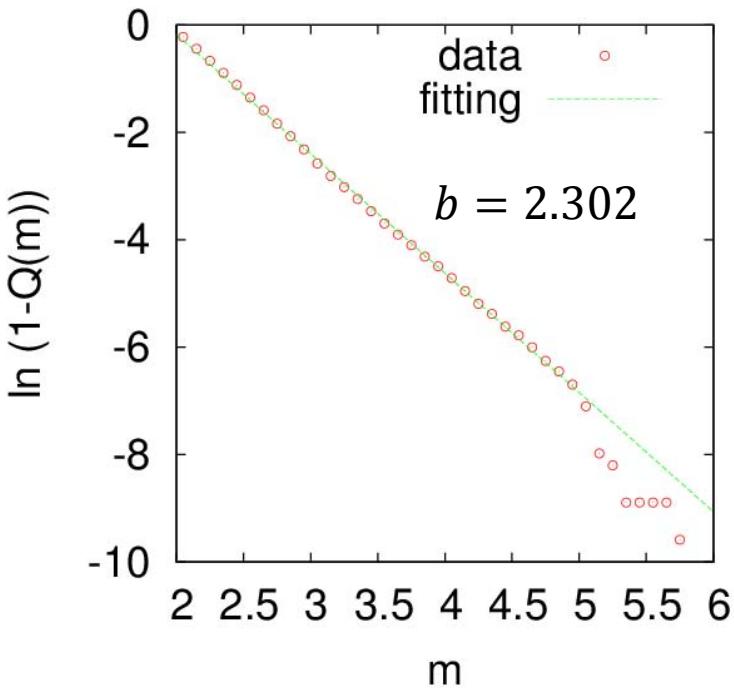


Empirical laws in seismicity statistics

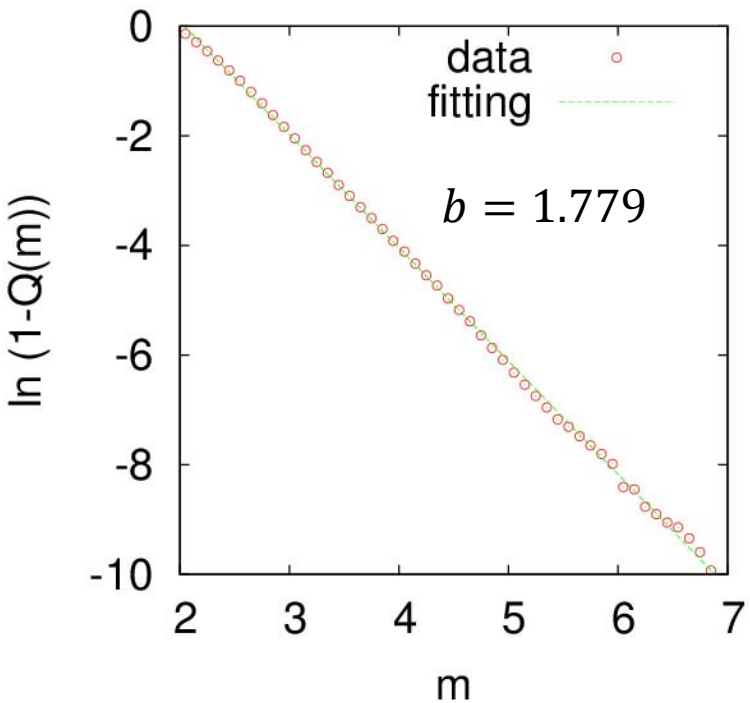
South California: 2001-2010, # *EQ=14605* ($m \geq 2.0$), SCEDC Catalog
Taiwan : 2001-2010, # *EQ=102985* ($m \geq 2.0$), TCWB Catalog

a) GR law: $Q(m) = 1 - e^{-bm}$ (in stationary regime)

South California



Taiwan

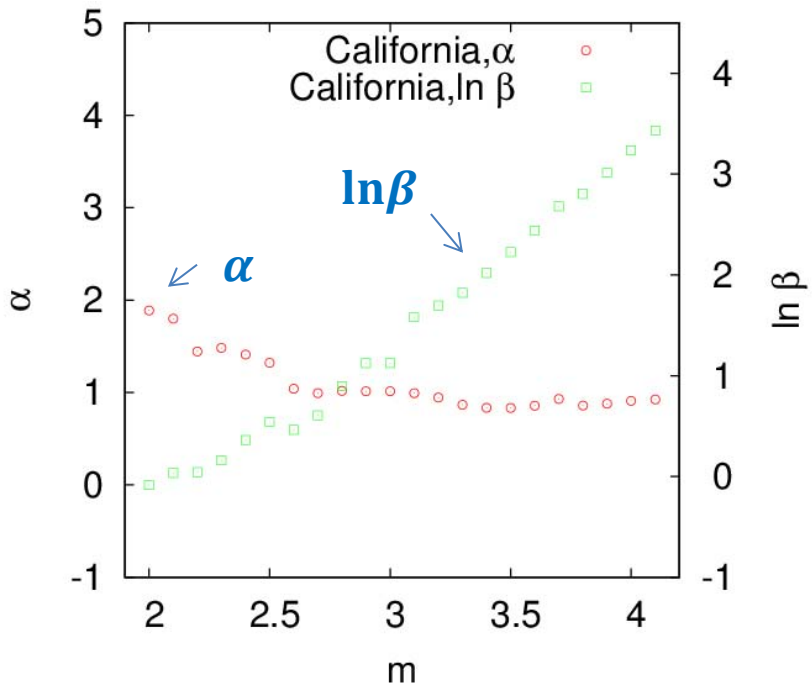


Empirical laws in seismicity statistics

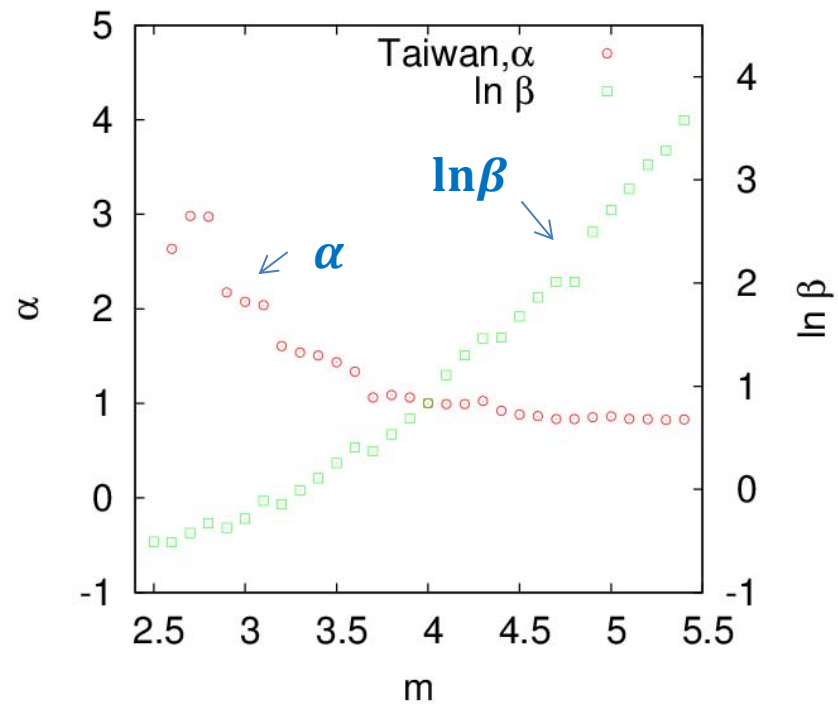
b) Multi-fractal relations :

$$P(\tau; m) = \frac{\alpha_m}{\beta_m}\left(\frac{\tau}{\beta_m}\right)^{\alpha_m - 1} exp[-\left(\frac{\tau}{\beta_m}\right)^{\alpha_m}]$$

South California



Taiwan



Summary of Part I

1. 3.11 EQs satisfy,

   i) Omori formula
   ii) GR formula
   iii) Weibull distribution for interoccurrence time

2. Multi-fractal relations are derived in each region ;

   i) Fore-foreshocks
   ii) Foreshocks
   iii) Aftershocks

$$\boxed{\alpha = f_\alpha(m, b), \qquad \beta = f_\beta(m, b)}$$

Part Ⅱ. Multi-fractal Relation and Unified Formula

(i). MF relation in stationary regime

(ii). MF relation in aftershock region

Theoretical Approaches (a universal constant)

Consider the case of natural EQs (JAPAN, California, Taiwan)
(in the stationary regime),

$$\begin{cases} P(m) = be^{-b(m-2)} & \text{(GR law)} \\ P(\tau; m) = \dfrac{\alpha_m}{\beta_m} e^{-\left(\frac{\tau}{\beta_m}\right)^{\alpha_m}} \left(\dfrac{\tau}{\beta_m}\right)^{\alpha_m - 1} & \text{(Weibull distribution)} \end{cases}$$

$$\Rightarrow \quad \boxed{\beta_m e^{-b(m-2)} \Gamma\left(1 + \frac{1}{\alpha_m}\right) = constant = e^{-k_{EQ}}}$$

(Ref. BusseiKenkyu, 2011)

$$\begin{cases} \alpha_m = f_\alpha(m, b) \\ \beta_m = f_\beta(m, b) \end{cases} \Longrightarrow b = g(\alpha, \beta)$$

Particular solution (near $\alpha = 1$, at $m = m_c$)

$$\begin{cases} \beta_m = \exp[b'(m - m_c) + C] \\ (b' - b)(m - m_c) \cong 0.43\Delta - 0.75\Delta^2, \qquad (\Delta = \alpha_m - 1) \end{cases}$$

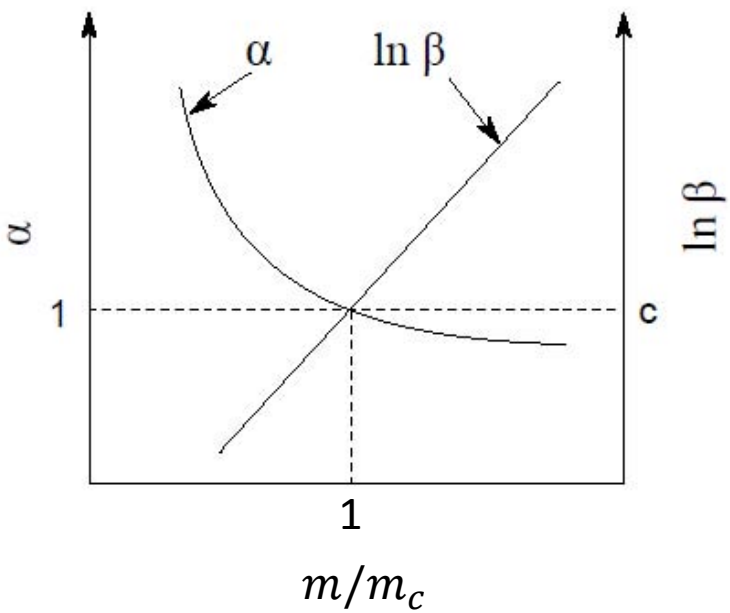
(perturbational expansion)

Prediction of the Next Big EQ with the magnitude m;

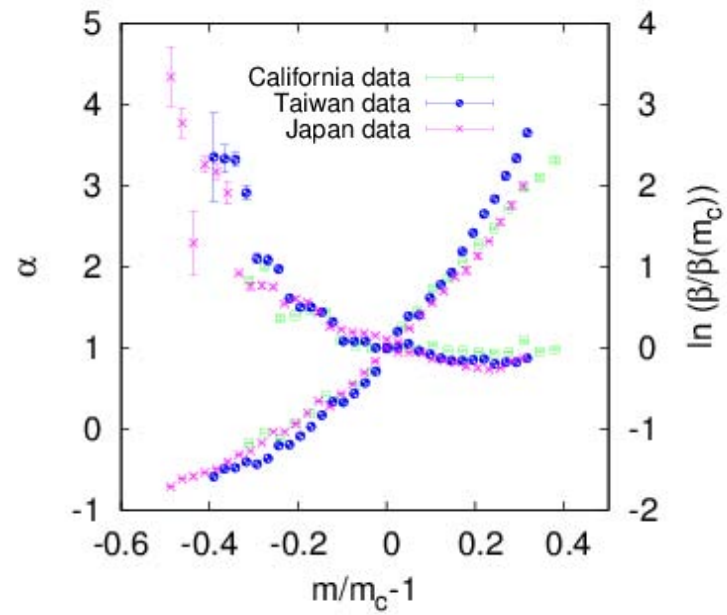
$$T = e^{-k_{EQ}} e^{b(m-2)} \quad (T\text{: waiting time for next EQ})$$

Theoretical Approaches (a universal constant)

Multi-fractal reation (theory)



Multi-fractal reation (for natural EQ)



California:
$m_c = 2.9$
$\beta(m_c) \fallingdotseq 0.97$
$b = 2.302$
Taiwan:
$m_c = 4.1$
$\beta(m_c) \fallingdotseq 1.08$
$b = 1.779$
Japan:
$m_c = 4.3$
$\beta(m_c) \fallingdotseq 1.17$
$b = 1.776$

Theoretical Approaches (Test of Universal Constant)

(i)Stationary regime (Japan, South California, Taiwan)

	Japan(Fore-foreshock) β_c: 1.17,m_c: 4.3	South California β_c: 0.97,m_c: 2.9	Taiwan β_c: 1.08,m_c: 4.1
m_0	k_{EQ}	k_{EQ}	k_{EQ}
3.4	2.12	1.11	2.49
3.5	2.11	1.13	2.51
3.6	2.19	1.17	2.52
3.7	2.15	1.20	2.68
3.8	2.20	1.26	2.70
3.9	2.21	1.30	2.72
4.0	2.36	1.32	2.72
4.1	2.37	1.37	2.62
4.2	2.37	-	2.61
4.3	2.36	-	2.64
4.4	2.34	-	2.76
4.5	2.31	-	2.71

(ii)Nonstationary regime (moving ensemble in aftershock region($t > 0$))

a)Generalized GR law

$$P_{GR}(m,t) = e^{-\hat{b}(m-2)} \frac{\ln(\frac{t+50+c_m}{t-50+c_m})}{\ln(\frac{t+50+c_{m_0}}{t-50+c_{m_0}})}$$

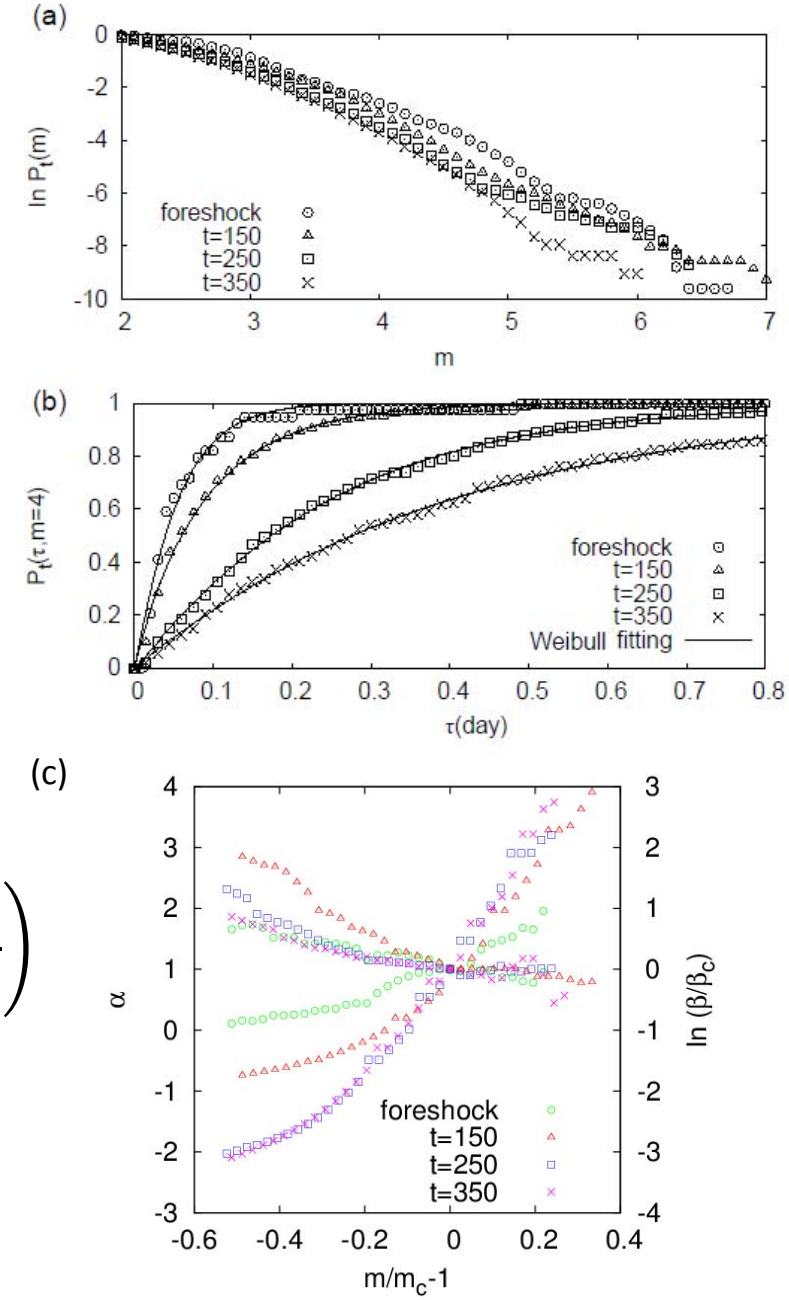
b) Weibull law for the interoccurrence time

$$P_W(\tau,m) = \exp[-\left(\frac{\tau}{\beta_m(t)}\right)^{\alpha_m(t)}]$$

c) Time independent MF universality

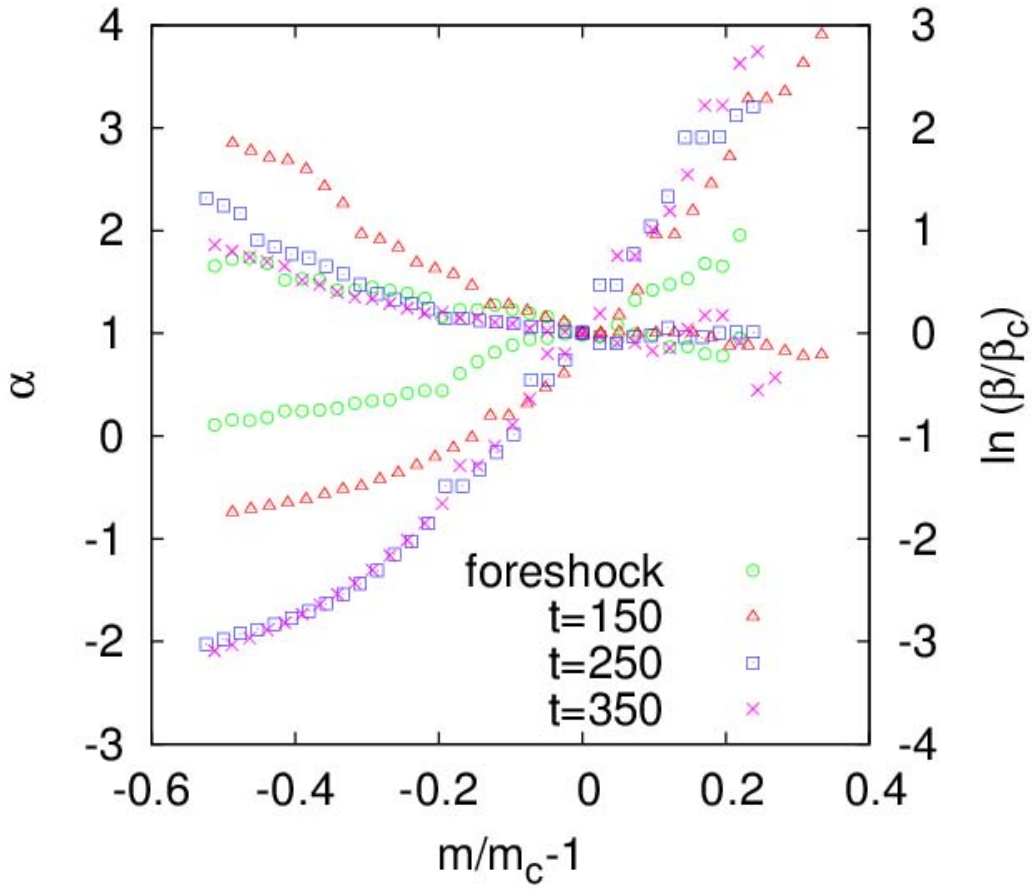
$$e^{-k_{EQ}(t)} = \beta_m(t) e^{-b(t)(m-2)} \Gamma\left(1 + \frac{1}{\alpha_m(t)}\right)$$

$$\Rightarrow \begin{cases} \alpha_m = f_\alpha(m, \hat{b}(t)) \\ \beta_m = f_\beta(m, \hat{b}(t)) \end{cases}$$



Theoretical Approaches (a universal constant)

Multi-fractal reation (for moving ensemble)



t=150:
$m_c = 3.9$
$\beta(m_c) \fallingdotseq 2.42$
$b = 1.21$
t=250 :
$m_c = 4.2$
$\beta(m_c) \fallingdotseq 0.94$
$b = 1.40$
t=350 :
$m_c = 4.1$
$\beta(m_c) \fallingdotseq 0.72$
$b = 1.50$

Theoretical Approaches (Test of Universal Constant)

(ii)Nonstationary regime (Japan; moving ensemble at t)

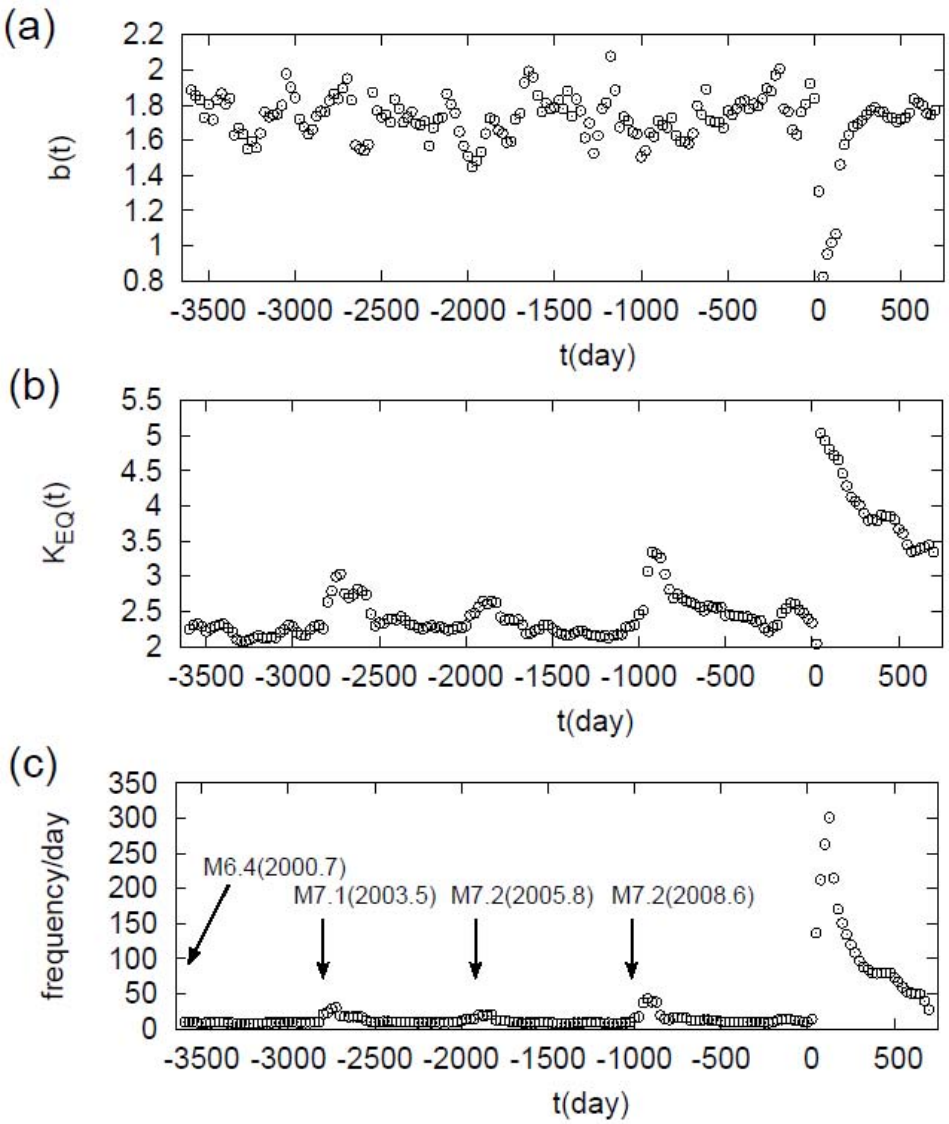
	t=150 β_c: 2.42,m_c: 3.9	t=250 β_c: 0.94,m_c: 4.2	t=350 β_c: 0.72,m_c: 4.1
m_0	k_{EQ}	k_{EQ}	k_{EQ}
3.4	5.00	4.43	4.17
3.5	5,12	4.57	4.32
3.6	5.11	4.54	4.26
3.7	5.07	4.52	4.21
3.8	5.04	4.48	4.08
3.9	4.72	4.08	3.79
4.0	4.84	4.22	3.94
4.1	4.80	4.14	3.88
4.2	4.67	4.02	3.83
4.3	4.25	3.64	3.38
4.4	4.37	3.77	3.53
4.5	4.26	3.65	3.37

(iii) Long-term fluctuations of statistical parameters. (moving ensembles)

a) Effective GR parameter ($\hat{b}$)

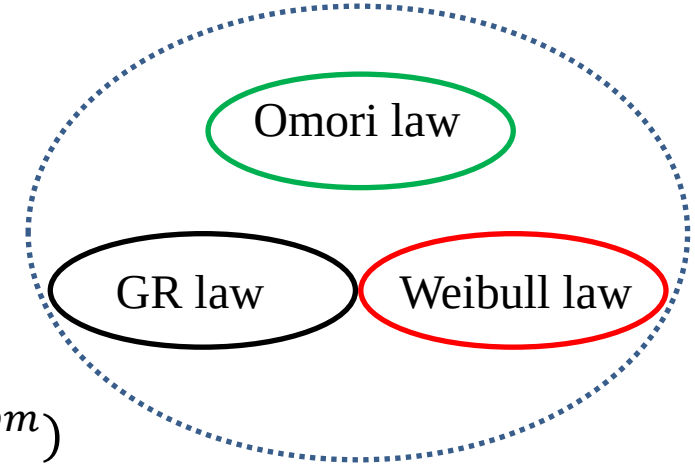
b) Earthquake constant (k_{EQ})

c) Shock frequency (per day)



Part Ⅲ. Conclusions

Summary



Unified formula in time-dependent form:

(i)aftershock frequency (per day)

$$n_m(t) = \frac{d}{1 + tc_m^{-1}}, (c_m = ae^{-bm})$$

(ii)magnitude distribution (cumulative prob.)

$$P_m(t) = \frac{n_m(t)}{n_{m_0}(t)} = e^{-b(m-m_0)} \frac{t + c_{m_0}}{t + c_m} \to e^{-b(m-m_0)}, (t \to \infty)$$

(iii)universal multi-fractal relation

$$\beta_m(t) n_m(t) \Gamma\left(1 + \frac{1}{\alpha_m(t)}\right) = 1$$

$$\to e^{-k_{EQ}(t)} = \beta_m(t) e^{-b(t)(m-m_0)} \Gamma\left(1 + \frac{1}{\alpha_m(t)}\right)$$

Prospects

Next challenge is the statistics in the prestage ($t \lesssim 0$).

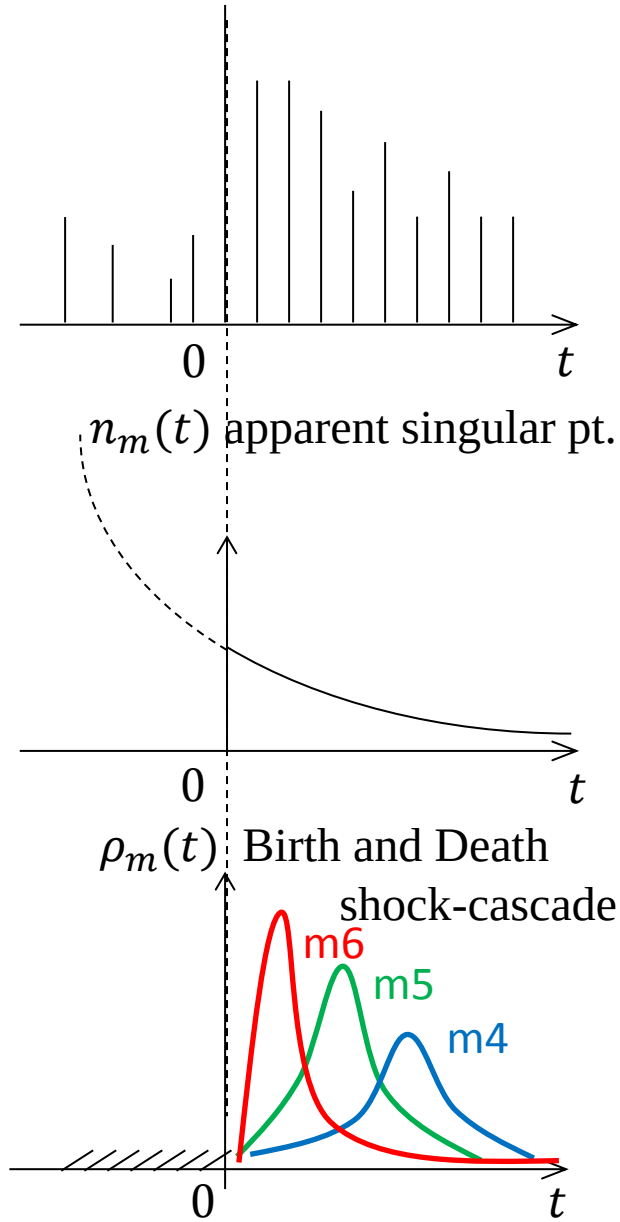
$$N_m(t) = dc_m \ln(1 + tc_m^{-1}),$$

Singular points : at $t = 0, N_m(0) = 0, n_m(0) = d$

$$n_m(t) = \frac{d}{1 + tc_m^{-1}} \quad \text{is not singular}$$

Magnitude density : $\rho_m(t) = \frac{dn_m(t)}{dt}$

What happened in the prestage : $t \leq 0$?



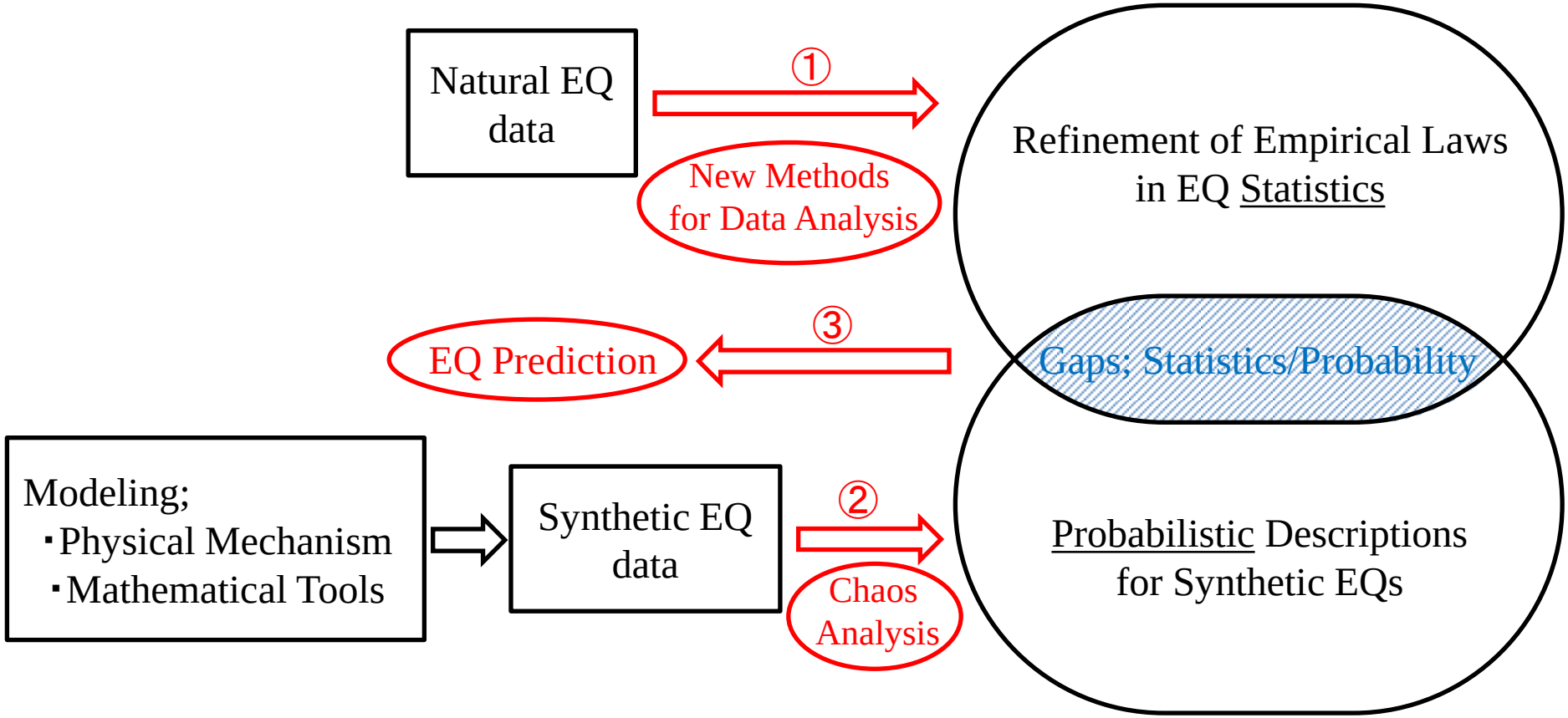
Members of our group working on EQ statistics ;

Mr. S. Tsugawa	(Student of Waseda University)
Mr. F. Saito	(Student of Waseda University)
Dr. T. Hasumi	(Mizuho Information and Research Institute)
Dr. T. Akimoto	(Keio University)
Prof. C-C. Chen	(National Central University, Taiwan)

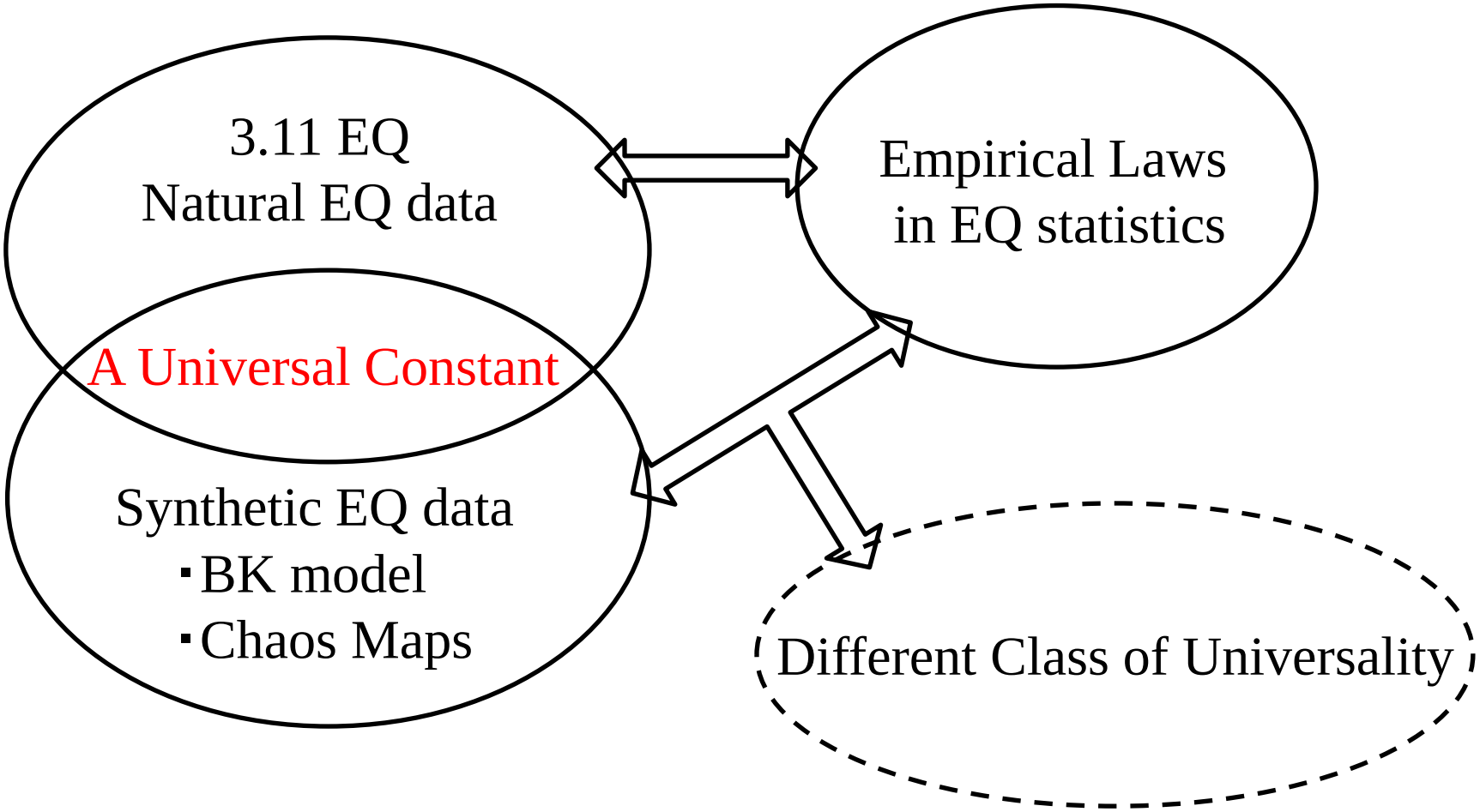
Reference

1. “Earthquake Reseach and Analysis”, (INTECH, 2012) Chap.1.
2. “Earthquakes ; Triggers, Environmental Impact and Potential Hazards”, (NOVAPUB-LISHERS, 2013) Chap.5.

Roles of Theoretical Studies on Earthquakes



Summary of My Talk

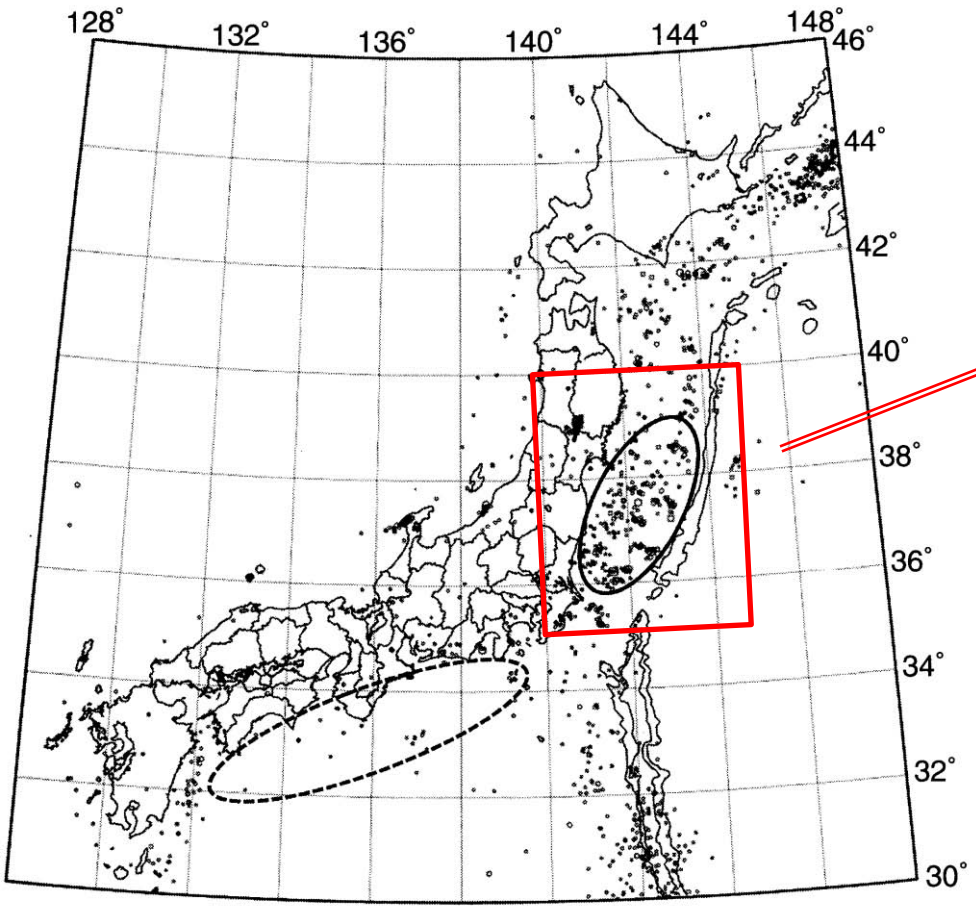


Part 0. Introduction: Outline of 3.11 EQ

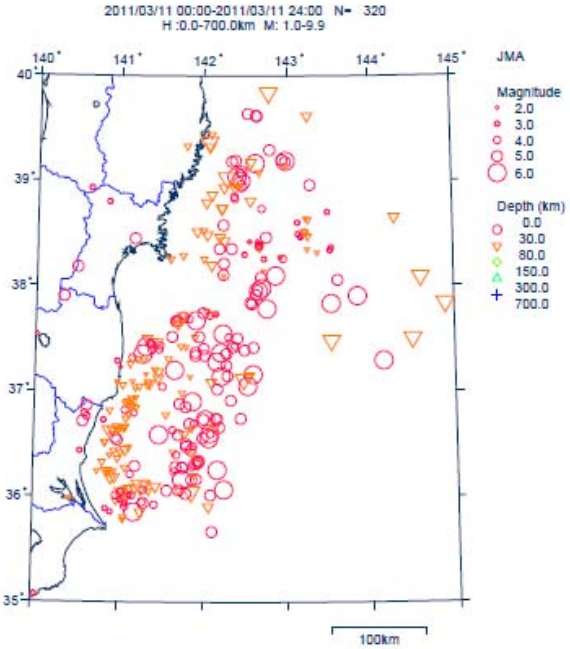
1. Platetectonics of Japanese Islands
2. Supercycle of gigantic EQs

EQs : 2006-2010

(Hypocenter, depth < 60km, M>4.0)



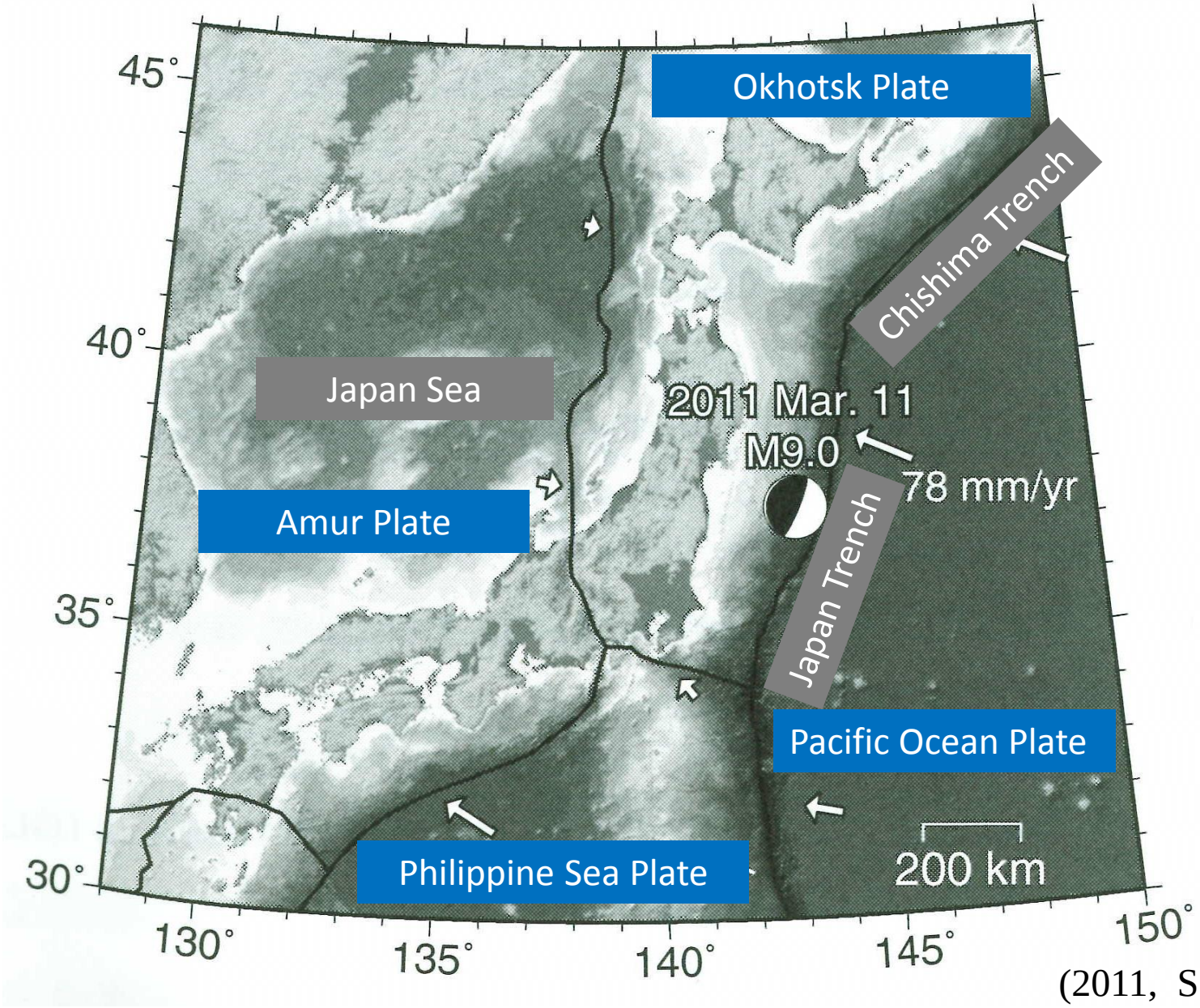
Aftershocks in Fukushima-Miyagi area
(3.11 – 11.11, 2011)



(JMA)

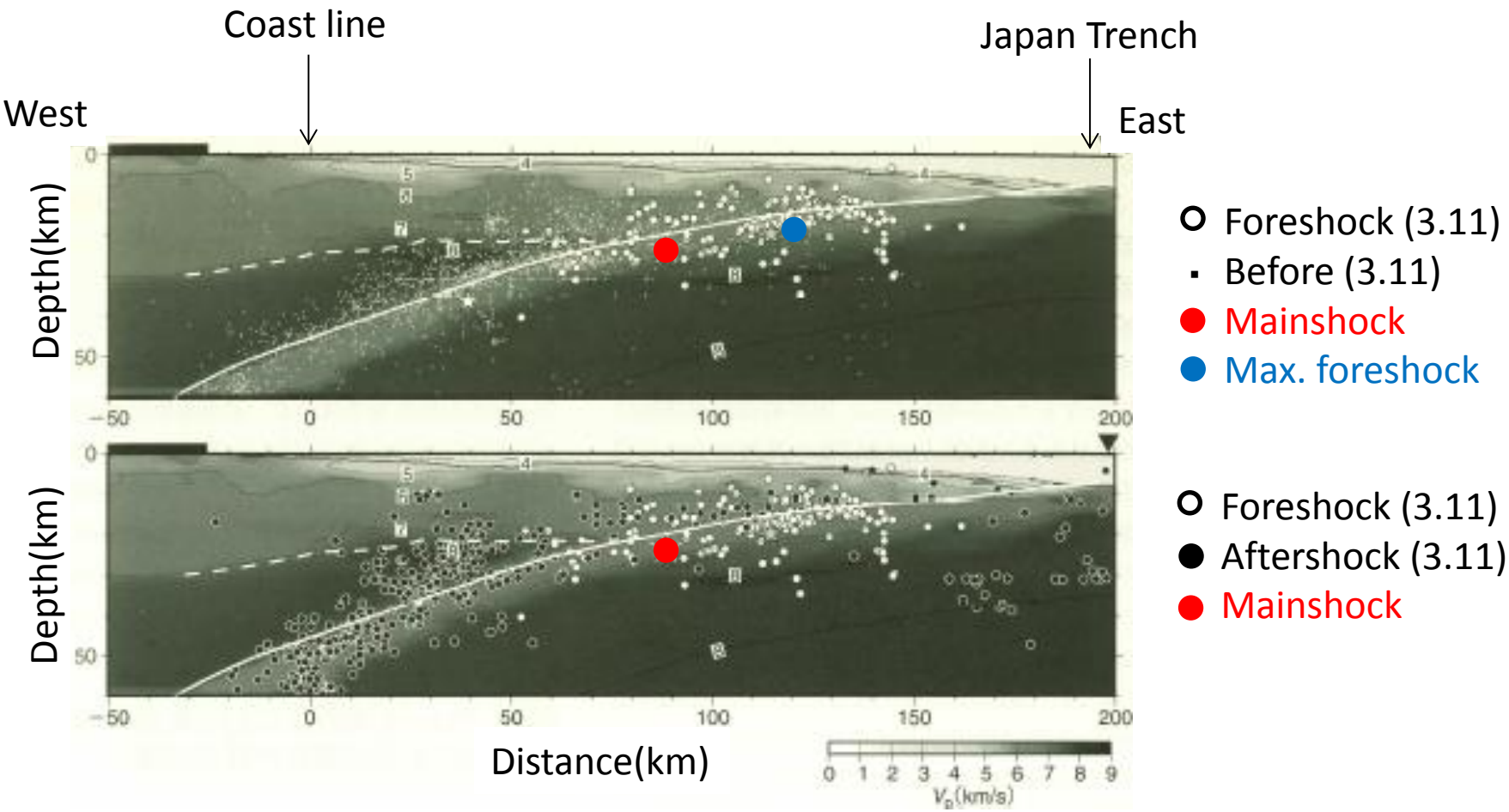
(2011, T. Matsuzawa)

Platetectonics



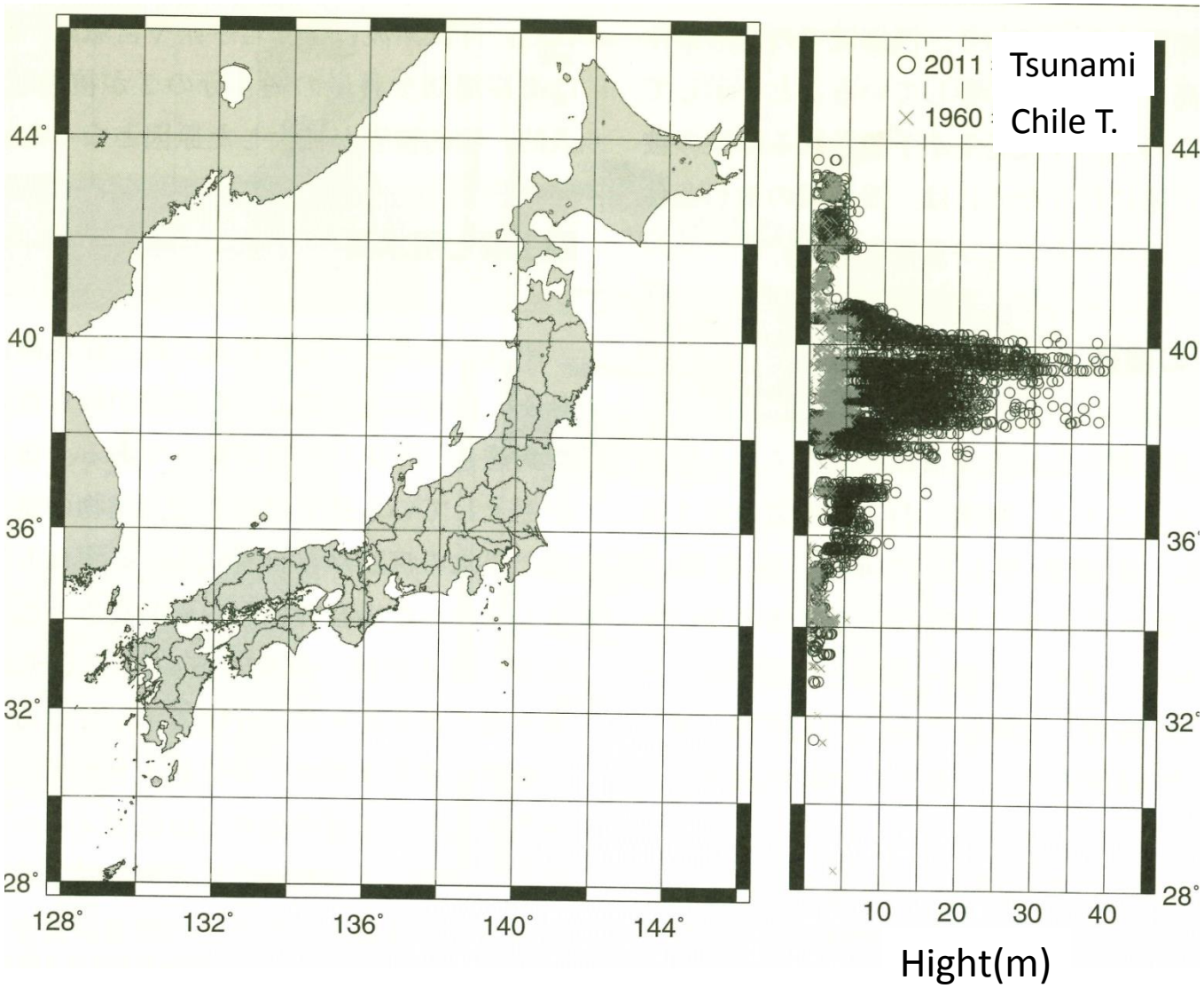
(2011, S. Ozawa)

Platetectonics(Cross-section)



(2011, R. Hino et. al.)

Hight of Tsunami



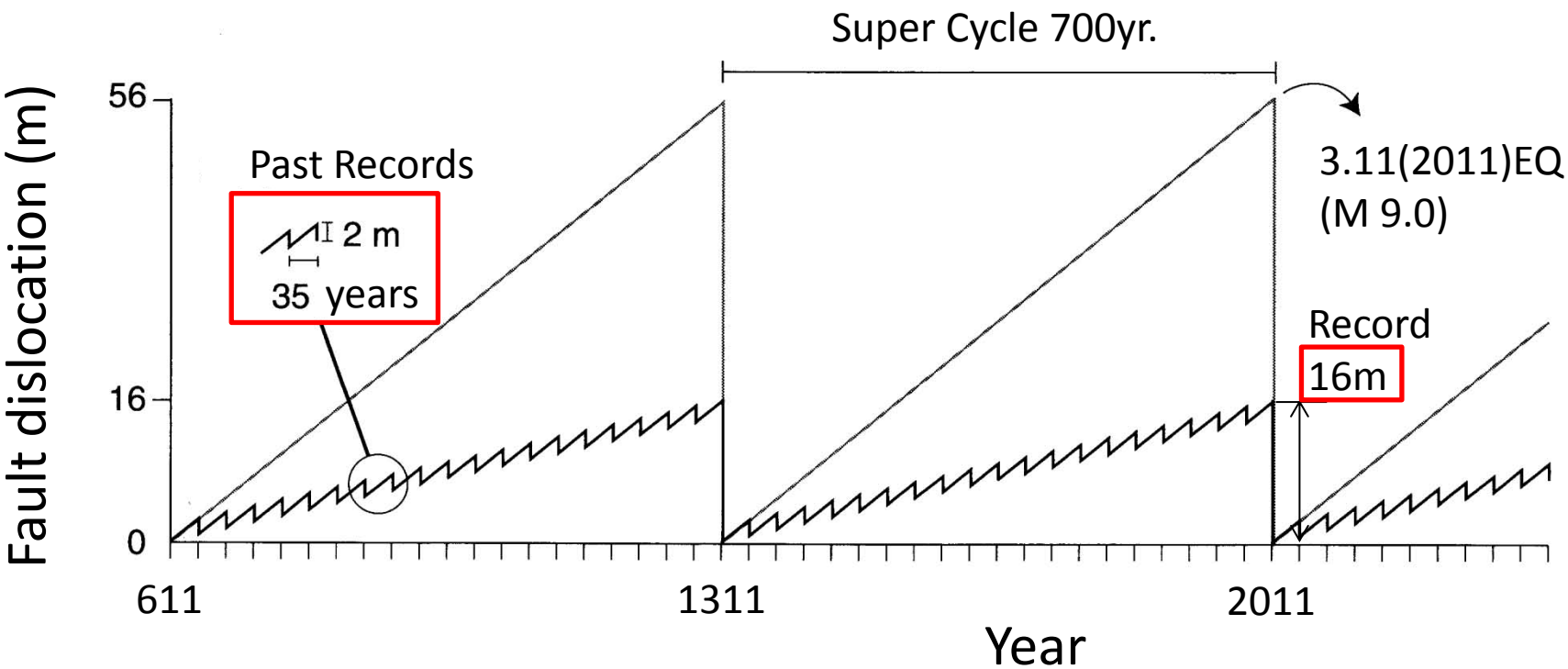
(2011, T. Takahashi)

南三陸町

Super cycles of gigantic EQs

$$T \times 0.08 = \frac{2}{35} \times T + 16 \ , \ \boxed{\text{Plate velocity} = 8\ (cm/year)}$$

$$\rightarrow T = 700\ (years)$$



(2011, K. Satake)